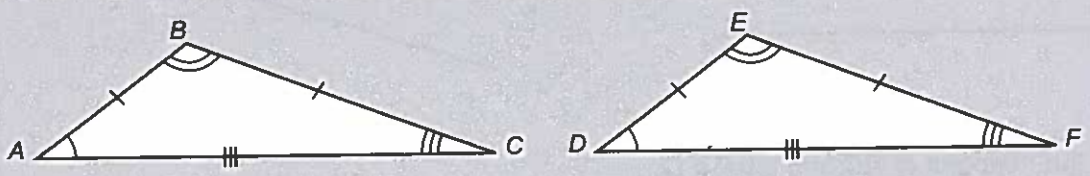
G8-30 Congruent Triangles

The corresponding sides and angles are marked in these triangles.



If the corresponding sides and angles in two polygons are equal, the two polygons are **congruent**. In ΔABC and ΔDEF above, all 3 pairs of sides and all 3 pairs of angles are equal.

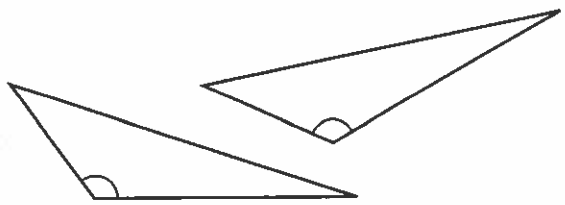
$AB = DE$ $BC = EF$ $AC = DF$ and $\angle A = \angle D$ $\angle B = \angle E$ $\angle C = \angle F$

So, ΔABC is congruent to ΔDEF. You can also write this as $\Delta ABC \cong \Delta DEF$.

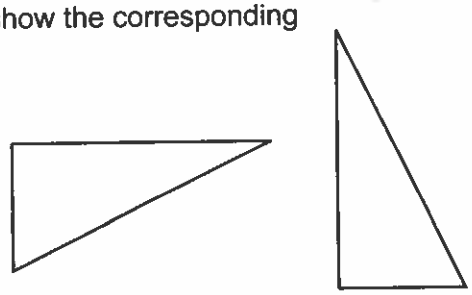
The reverse is also true: If two polygons are congruent, all the corresponding sides and angles are equal.

1. The two triangles are congruent. Mark the triangles to show the corresponding equal angles.

a)

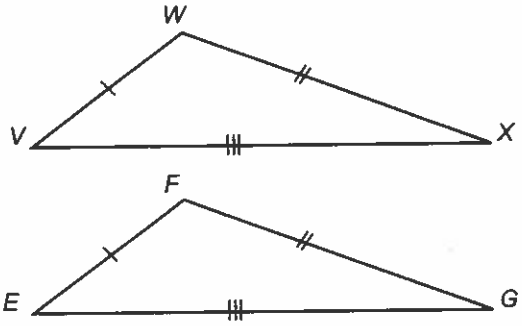


b)



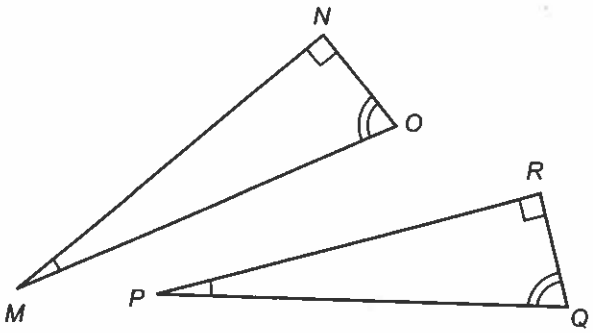
2. Which sides in each pair of triangles are corresponding equal sides?

$VW =$ ___ $WX =$ ___ $VX =$ ___



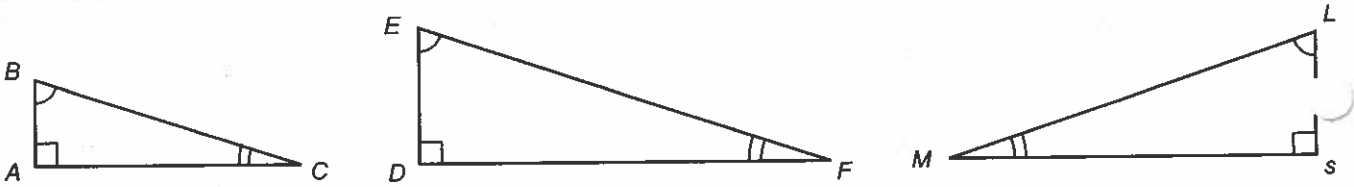
3. Which angles in each pair of triangles are corresponding equal angles?

$\angle M = \angle$___ $\angle N = \angle$___ $\angle O = \angle$ ___

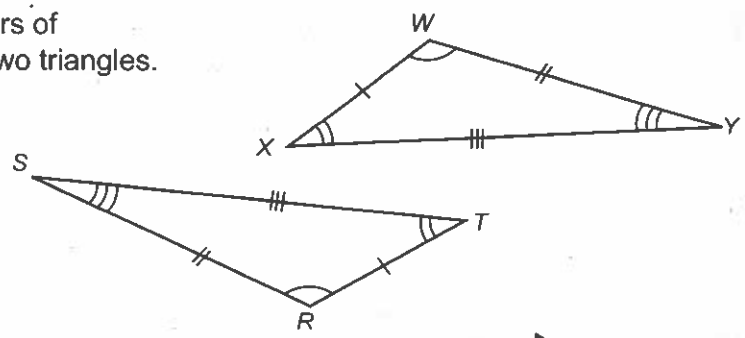


4. These triangles each have the same three angles. Which angles are corresponding equal angles?

$\angle A = \angle$___ $= \angle$___ $\angle B = \angle$___ $= \angle$___ $\angle C = \angle$___ $= \angle$___



5. These triangles are congruent. Identify the pairs of corresponding equal sides and angles in the two triangles.



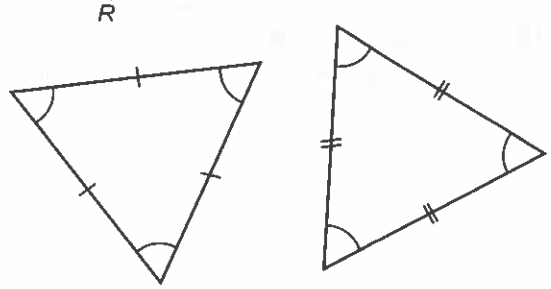
$\angle$ ____ = $\angle$ ____ $\angle$ ____ = $\angle$ ____

$\angle$ ____ = $\angle$ ____

_____ = _____ _____ = _____

_____ = _____

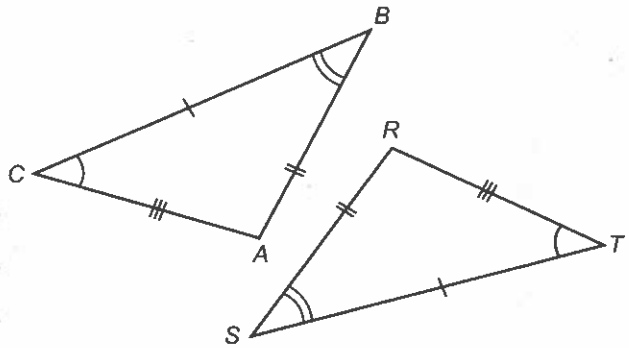
6. Kali notices that there are three equal angles in these two triangles. She decides that the triangles are congruent. Is she right? What did she forget to check?



_____

_____

7. These two triangles are congruent. Why do you not have to mark the third pair of angles to know that they are equal? (Hint: If you know the measures of two angles in a triangle, then you know the measure of the third angle. Why?)



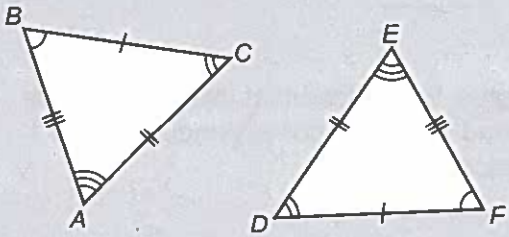
_____

_____

_____

Important: When two triangles are congruent, you have to write the letters that name the vertices in the proper order to show what is equal to what.

Example: The markings show that these two triangles are congruent. To write a correct congruence statement, follow these steps.



First triangle	Second triangle	Congruence statement
Choose any vertex on one of the triangles, e.g., vertex *A*.	Record the corresponding vertex on the other triangle, e.g., $\angle A = \angle E$	$\Delta A \cong \Delta E$
Choose an adjacent vertex, e.g., vertex *B*.	Record the corresponding vertex on the other triangle.	$\Delta AB \cong \Delta EF$
Write the letter of the third vertex.	Write the letter of the third vertex.	$\Delta ABC \cong \Delta EFD$

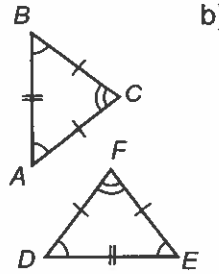
. Write a correct congruence statement for the pairs of triangles in Questions 5 and 7.

5. Δ ____ $\cong$ Δ ____ 7. Δ ____ $\cong$ Δ ____

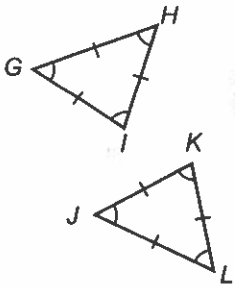
9. There are 2 or more equal angles in these congruent triangles. For each pair of triangles:

i) Write the pairs of equal angles.
ii) Write a congruence statement for the triangles.
iii) Write a different congruence statement that is also correct.

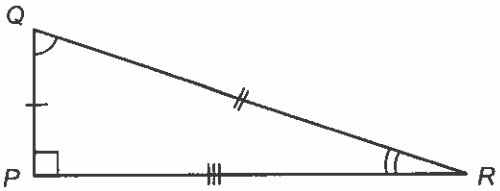
a)

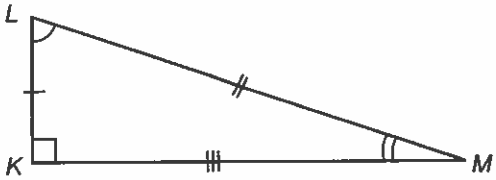


b)



10. A correct congruence statement also tells you which sides are equal. Finish circling the corresponding sides and angles in congruent triangles *PQR* and *KLM*.





Corresponding sides

$\triangle PQR = \triangle KLM$

$\triangle PQR = \triangle KLM$

$\triangle PQR = \triangle KLM$

Corresponding angles

$\triangle PQR = \triangle KLM$

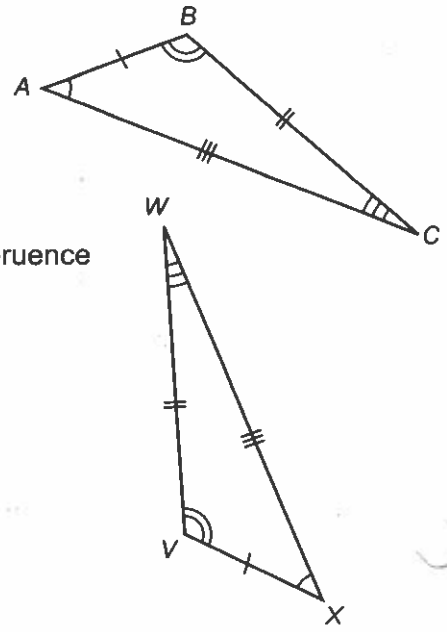
$\triangle PQR = \triangle KLM$

$\triangle PQR = \triangle KLM$

11. $\triangle UVW \cong \triangle XYZ$. Use the order of the vertex names in the congruence statement to write the list of corresponding sides and angles for these two congruent triangles.

$UV = XY$	$\angle U = \angle X$
$VW = YZ$	$\angle \_\_\_ = \angle \_\_\_$
$\_\_\_ = \_\_\_$	$\angle \_\_\_ = \angle \_\_\_$

12. Tom looked at these two triangles. He noticed that there are 3 pairs of corresponding equal sides and 3 pairs of corresponding equal angles. He wrote this congruence statement:



$\triangle VWX \cong \triangle ABC$

Then Tom lost his diagram, but he was not worried. He used his congruence statement to list the equal sides and angles. Here is his list:

$VW = AB$ $\angle V = \angle A$
$WX = BC$ $\angle W = \angle B$
$XV = CA$ $\angle X = \angle C$

Compare Tom's list of equal sides and angles with the diagrams. Is his list correct? Put an X beside any statements that are incorrect.

How should Tom have written the congruence statement?

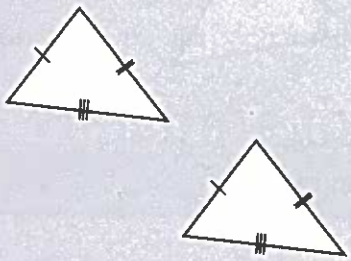
$\triangle$ _____ $\cong \triangle$ _____

G8-31 Congruence Rules

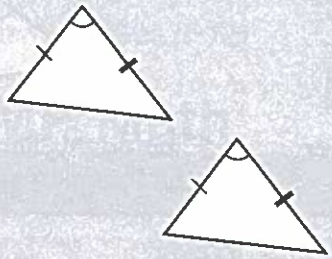
Congruence Rules for Triangles

Here are three rules for determining if two triangles are congruent:

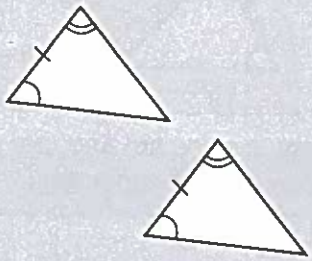
SSS (side-side-side)



SAS (side-angle-side)



ASA (angle-side-angle)

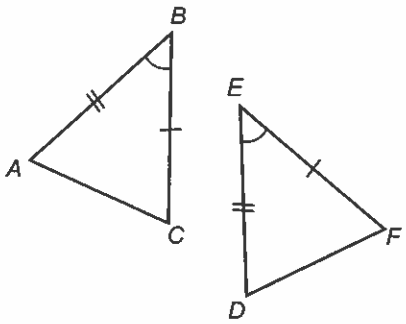


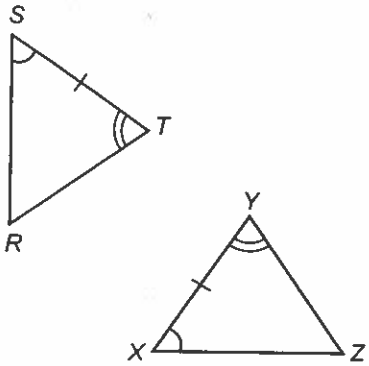
If two triangles have...

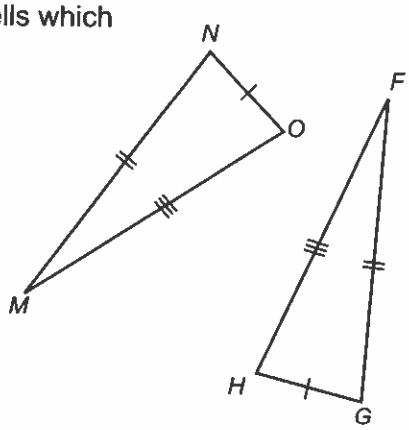
- three pairs of equal corresponding sides
- two pairs of equal corresponding sides and a pair of equal corresponding angles between these sides
- two pairs of equal corresponding angles and a pair of equal corresponding sides between them

...then the triangles are congruent.

1. Which congruence rule tells that the two triangles are congruent? Write the vertex letters in the names of the triangles in the correct order (the order that tells which corresponding parts are equal).







Congruence rule: _____

ΔABC is congruent to Δ _____.

Congruence rule: _____

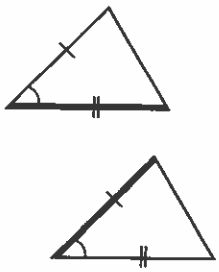
Δ ____ is congruent to Δ _____.

Congruence rule: _____

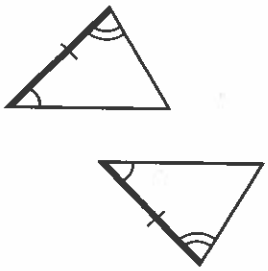
Δ _____ $\cong$ Δ _____.

2. Are the two thick sides corresponding sides? Explain.

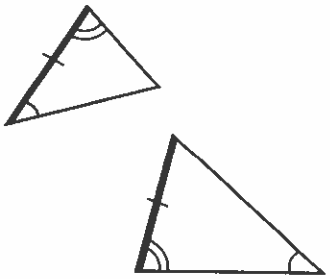
a)



b)

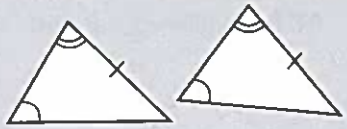


c)

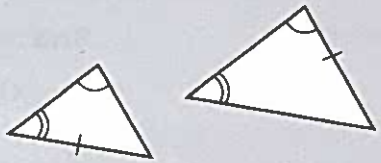


The fourth congruence rule for triangles is **AAS** (angle-angle-side):

If two triangles have two pairs of equal corresponding angles and one pair of equal corresponding sides that are opposite equal angles then the triangles are congruent.



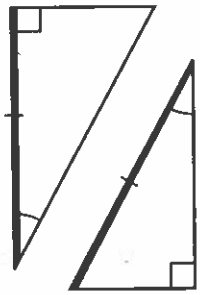
The equal sides are opposite equal angles — the triangles are congruent.



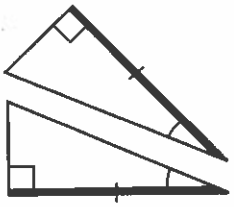
The equal sides are opposite different angles — the triangles are **not** congruent.

3. Are the two thick sides corresponding? Are they equal?

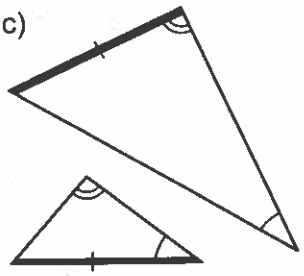
a)



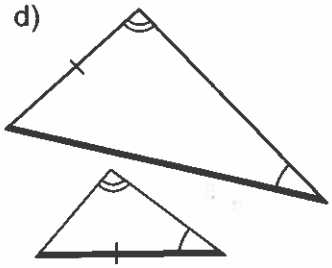
b)



c)



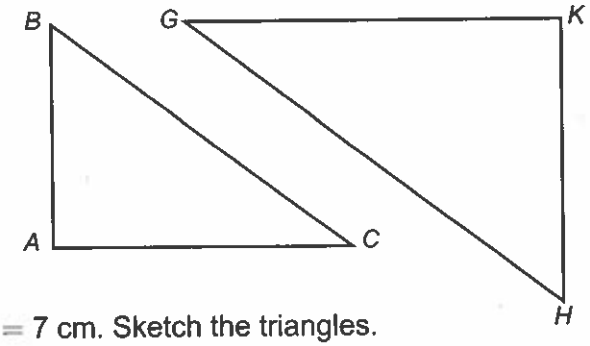
d)



4. Use each of the four congruence rules to draw a pair of congruent triangles. Name each triangle by choosing a letter for each vertex. Mark each pair of triangles to show the corresponding angles and sides.

5. Draw any triangle. Then draw another triangle that is not congruent to the first triangle. Explain why the triangles are not congruent.

6. Measure the angles and the sides of these triangles.
Write the measurements on the diagram.
Which angles are equal? Which sides are equal?
Are the triangles congruent? Explain.



7. $\triangle PQR$ and $\triangle XYZ$ have $PQ = XY = 5$ cm and $QR = YZ = 7$ cm. Sketch the triangles.
Will each extra condition make $\triangle PQR$ and $\triangle XYZ$ congruent? By which congruence rule?

a) $\angle P = \angle Y$ b) $\angle Q = \angle Y$ c) $\angle P = \angle X$ d) $PR = XZ$

8. $\triangle BCD$ and $\triangle FGH$ have $PQ = XY = 5$cm and $QR = YZ = 7$cm. Sketch the triangles.
Will each extra condition make the triangles congruent? By which congruence rule?

a) $\angle B = \angle F$ b) $CD = GH$ c) $BD = FH$ d) $BC = GH$

9. Sketch a counter-example to show why each statement is **false**.

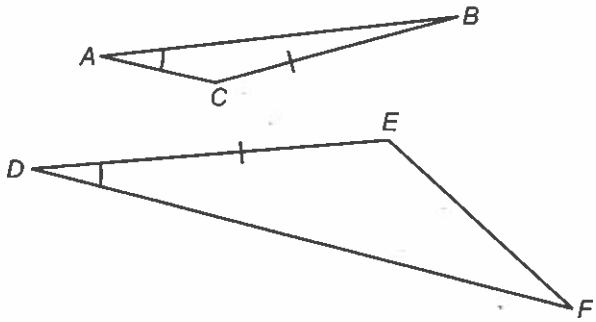
a) If two triangles have two corresponding sides that are equal, the triangles are congruent.

b) If two triangles have three equal parts (sides or angles), then they are congruent.

10. ΔABC and ΔDEF are both isosceles triangles. $\angle A = \angle D$ and $AB = DE$. Are ΔABC and ΔDEF always congruent? Explain.

Hint: Start by making a sketch that includes all the information you have been given. Try making more than one sketch using a different position for the equal angles.

INVESTIGATION ▶ Are two triangles that have two pairs of corresponding sides and one pair of corresponding angles **always** congruent?



A. What sides and angles in ΔABC and ΔDEF are equal?

_____

B. Can you use the SAS congruence rule? Why or why not?

_____

C. Do triangles ΔABC and ΔDEF look congruent? _____

D. Is **SSA** (side-side-angle) a congruence rule? What about **ASS** (angle-side-side)? Explain.

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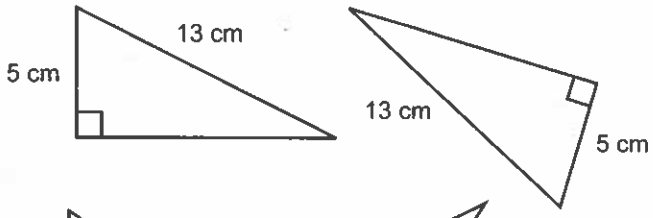
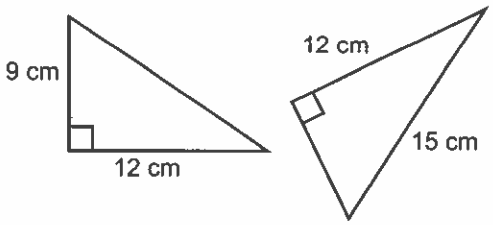
11. Explain why these two triangles are congruent. Which theorem will you use? Which congruence rule? (SAS, ASA, SSS or AAS)

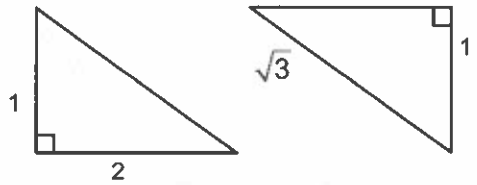


12. Explain why these two triangles are congruent. Which theorem will you use? Which congruence rule? (SAS, ASA, SSS or AAS)

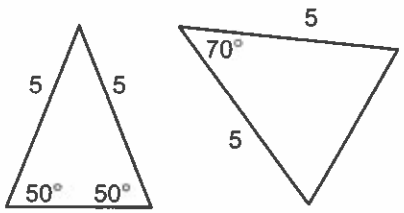


13. Are these two triangles congruent? How do you know? Note: Triangles are not drawn to scale.

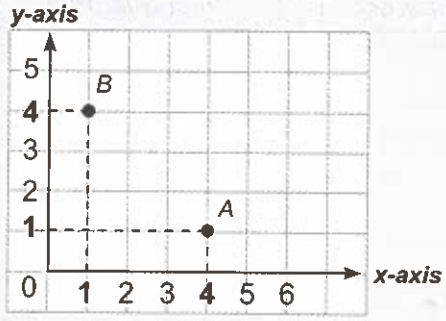
a)



b)



G8-8 Introduction to Coordinate Systems



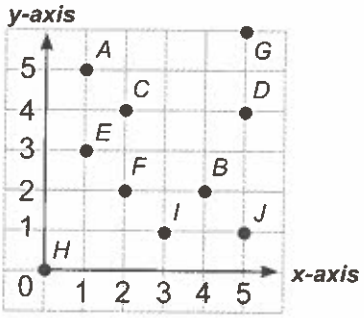
The position of a point on a coordinate grid is identified by an ordered pair of numbers in a bracket.

A (4, 1) — *x-coordinate* → 4, *y-coordinate* → 1

B (1, 4) — *x-coordinate* → 1, *y-coordinate* → 4

The point (0, 0) is called the **origin**.

1. Fill in the coordinates for the given points.



A (1 , 5) *B* (__, __)

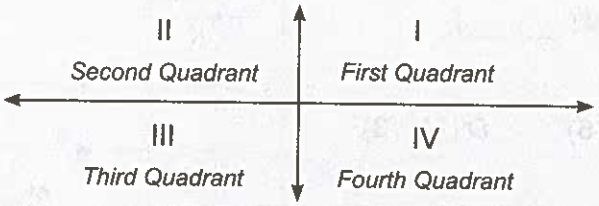
C (__, __) *D* (__, __)

E (__, __) *F* (__, __)

G (__, __) *H* (__, __)

I (__, __) *J* (__, __)

A grid that has been extended to include negative integers is called a **Cartesian coordinate system**. We use Roman numerals to number the quadrants:
1 = I, 2 = II, 3 = III, 4 = IV.



2. a) Label the origin (*O*) and the *x*- and *y*-axes.

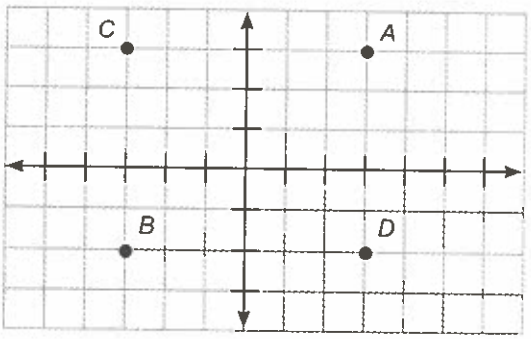
b) Label the axes with positive and negative integers.

c) Number the four quadrants (using I, II, III, IV).

d) Which quadrants are these points in?

A (3, 3) _I_ *B* (–3, –2) ___

C (–3, 3) ___ *D* (3, –2) ___



3. In Figure 1, point A (2, 3) is in the first quadrant. Its x- and y-coordinates are both **positive**.

   a) Find the coordinates of points…

   P (___, ___) Q (___, ___)

   R (___, ___) S (___, ___)

   b) Plot and label.

   B (3, 2) C (1, 4) D (4, 1)

4. In Figure 1, point F (−2, 3) is in the second quadrant. Its x-coordinate is **negative** and its y-coordinate is **positive**.

   a) Find the coordinates of points…

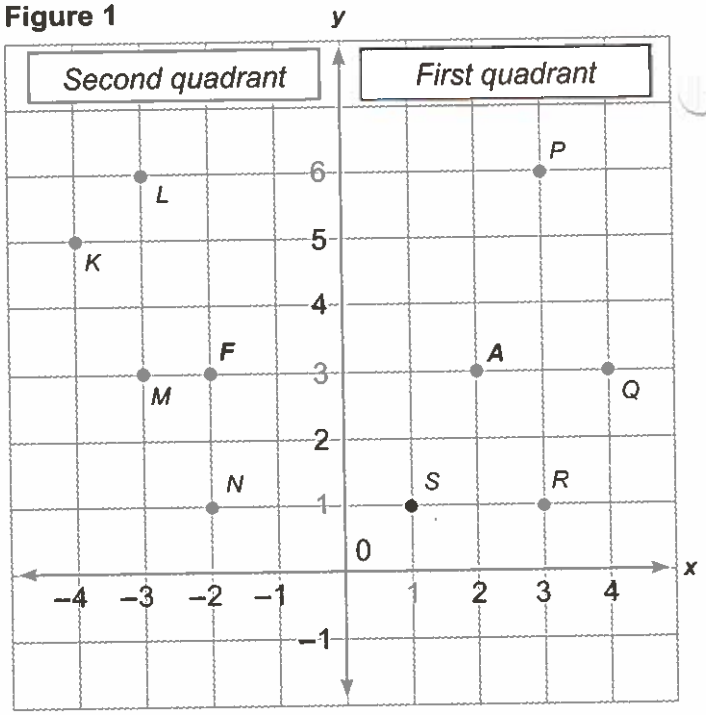
   K (___, ___) L (___, ___)

   M (___, ___) N (___, ___)

   b) Plot and label.

   G (−3, 2) H (−1, 6) I (−4, 1)

Figure 1



5. In Figure 2, point A (−2, −3) is in the third quadrant. Its x- and y-coordinates are both **negative**.

   a) Find the coordinates of points…

   K (___, ___) L (___, ___)

   M (___, ___) N (___, ___)

   b) Plot and label.

   B (−3, −4) C (−2, −6) D (−4, −3)

6. In Figure 2, point F (2, −3) is in the fourth quadrant. Its x-coordinate is **positive** and its y-coordinate is **negative**.

   a) Find the coordinates of points…

   P (___, ___) Q (___, ___)

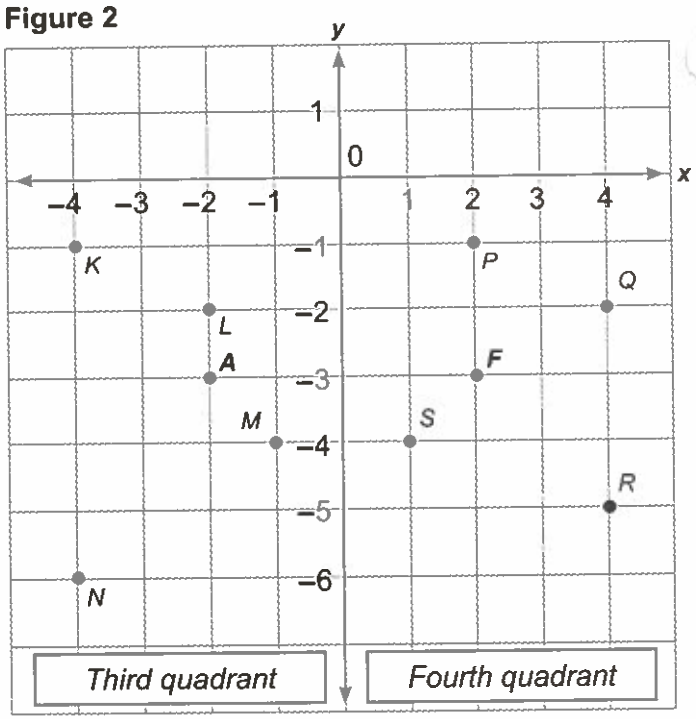
   R (___, ___) S (___, ___)

   b) Plot and label.

   G (3, −4) H (1, −6)

   I (4, −1) J (1, −2)

Figure 2



7. In Figure 3, points B (2, 0) and C (−4, 0) are both on the x-axis. The y-coordinate of any point on the x-axis is **zero**.

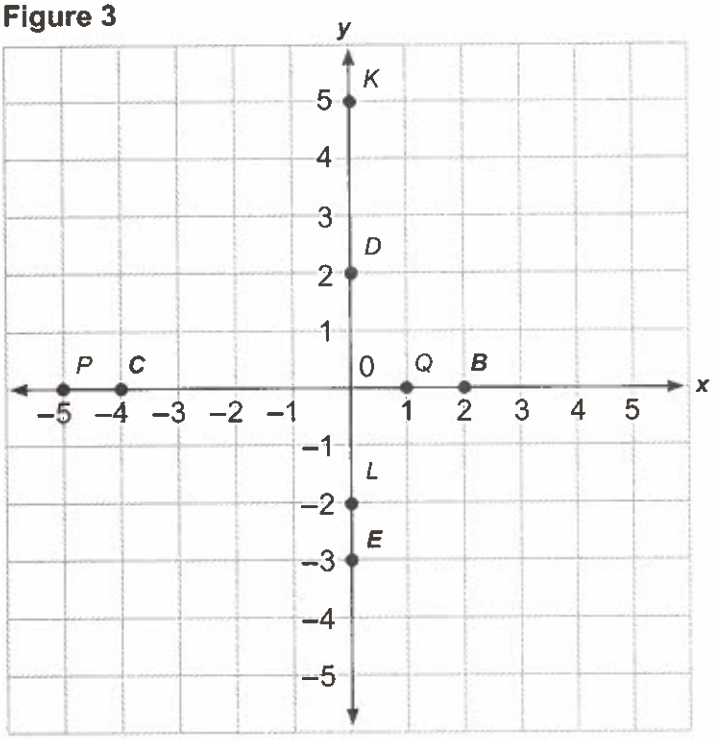
a) Find the coordinates of points…

P (___, ___) Q (___, ___)

b) Plot and label.

A (3, 0) M (−3, 0)

Figure 3



8. In Figure 3, points D (0, 2) and E (0, −3) are both on the y-axis. The x-coordinate of any point on the y-axis is **zero**.

a) Plot and label.

G (0, 4) H (0, −1)

b) Find the coordinates of points…

K (___, ___) L (___, ___)

9. a) In Figure 4, find the coordinates of points…

P (___, ___) Q (___, ___) R (___, ___) S (___, ___)

T (___, ___) U (___, ___) V (___, ___) W (___, ___)

b) Plot and label these points in Figure 4.

A (3, 4) B (5, −2)

C (−3, −2) D (−4, 1)

E (3, 0) F (0, 2)

G (0, 3) H (−5, 0)

c) Sort the points in Figure 4 by location.

first quadrant: A, Q

second quadrant: _____

third quadrant: _____

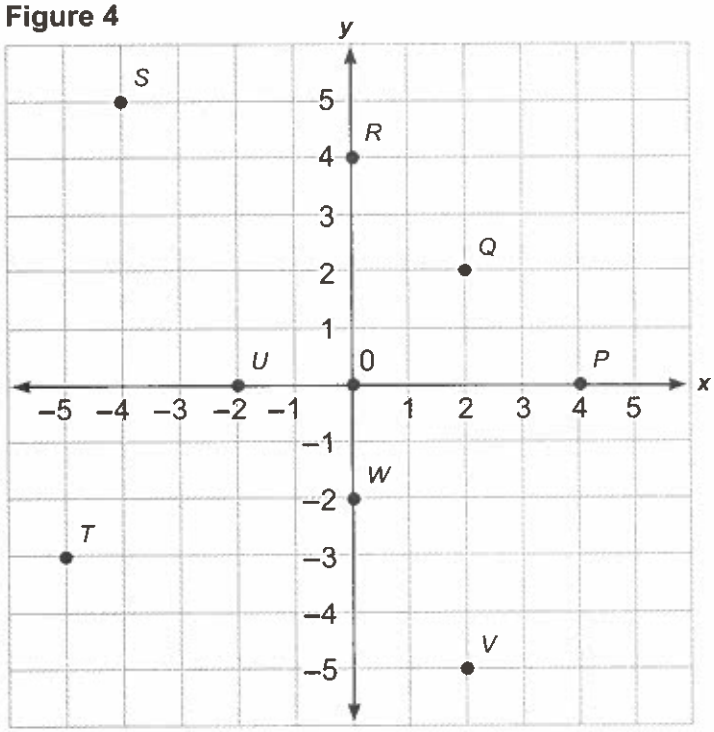
fourth quadrant: _____

on the x-axis: _____

on the y-axis: _____

at the origin: _____

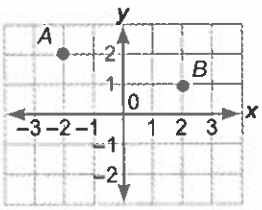
Figure 4



G8-9 Translations

1. How many units right or left and how many units up or down did the dot slide from position *A* to *B*?

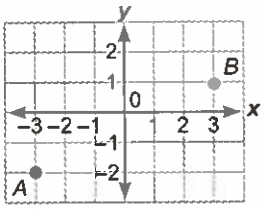
a)



_4_ units right, _1_ unit down

A (_2_, _2_) *B* (_2_, _1_)

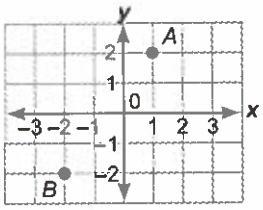
b)



___ units right, ___ units up

A (___, ___) *B* (___, ___)

c)

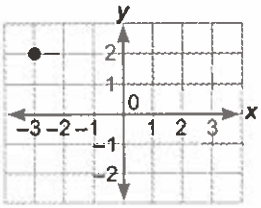


___ units left, ___ units down

A (___, ___) *B* (___, ___)

2. Slide the point by the given number of units. The resulting point is called the **image**.

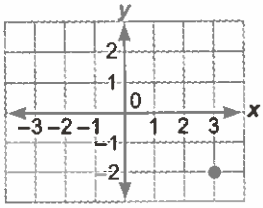
a) 5 units right; 2 units down



original point (___, ___)

image (___, ___)

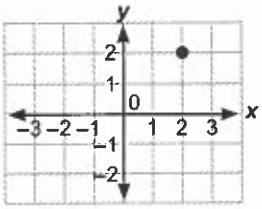
b) 6 units left; 3 units up



original point (___, ___)

image (___, ___)

c) 5 units left; 4 units down

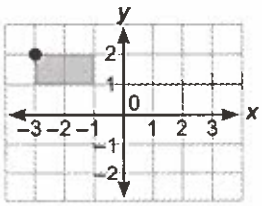


original point (___, ___)

image (___, ___)

3. Slide the point two units down, then copy the shape. Write the coordinates of the point and its image.

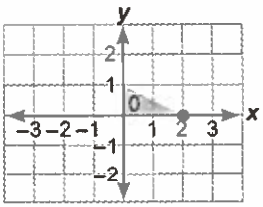
a)



original point (___, ___)

image (___, ___)

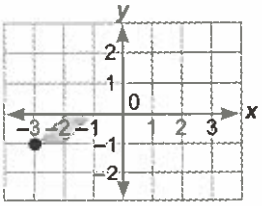
b)



original point (___, ___)

image (___, ___)

c)

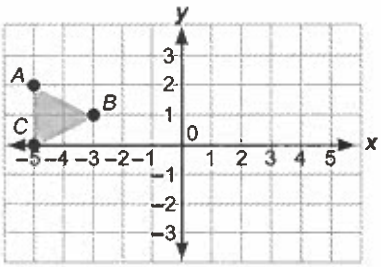


original point (___, ___)

image (___, ___)

4. Slide each triangle 5 units to the right and 3 units down.

a)

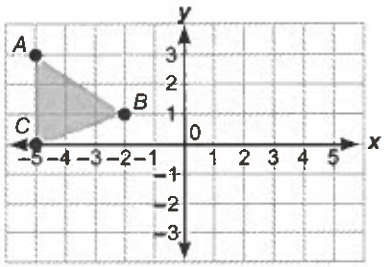


A (___, ___) → *A'* (___, ___)

B (___, ___) → *B'* (___, ___)

C (___, ___) → *C'* (___, ___)

b)



A (___, ___) → *A'* (___, ___)

B (___, ___) → *B'* (___, ___)

C (___, ___) → *C'* (___, ___)

You can show how a point moves with an arrow: *A* → *A'*

5. a) Describe how point D moved to point D':

4 units right , _____ units _____

b) Draw an arrow to show where point A moved to under the translation.

c) Describe how point A moved:

___ units _____, _____ units _____

d) Did all of the points on the parallelogram move the same amount right and the same amount up? _____

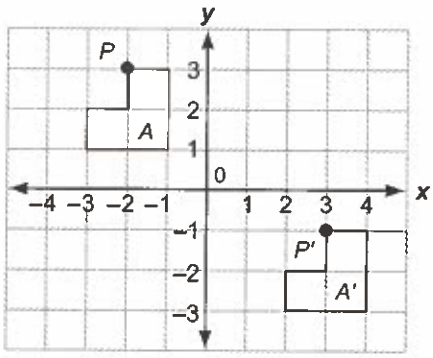
e) Fill in the coordinates of the vertices of the original parallelogram $ABCD$ and the image $A'B'C'D'$.

A (___, ___) → A' (___, ___) B (___, ___) → B' (___, ___)

C (___, ___) → C' (___, ___) D (___, ___) → D' (___, ___)

6. Draw a translation arrow from vertex P of shape A to the corresponding vertex P' in A'. Then write the coordinates of P and P'.

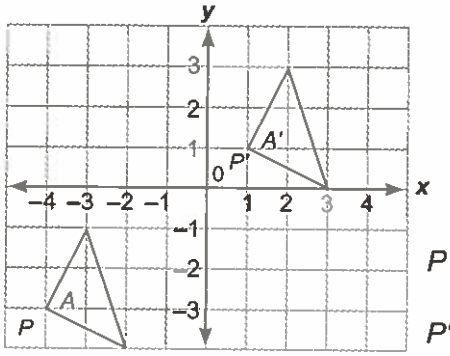
a)



P (___, ___)

P' (___, ___)

b)

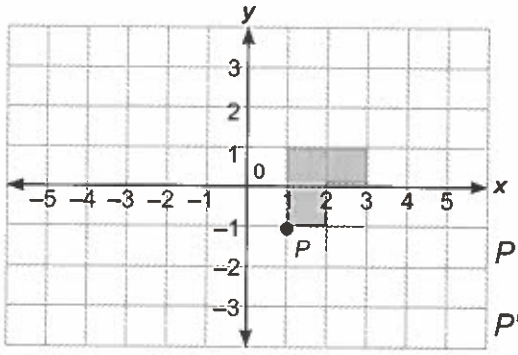


P (___, ___)

P' (___, ___)

7. Slide the shapes in the grids below using a translation of your choice. Describe how far the shape moved (right/left and up/down) and write the coordinates of P and P'.

a)

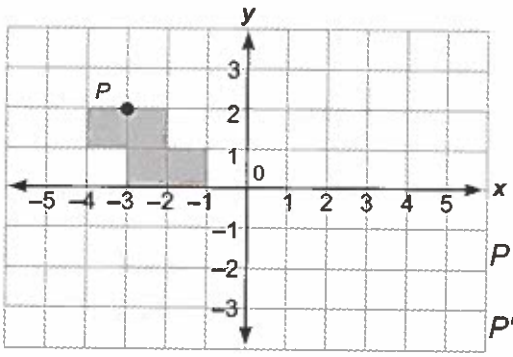


P (___, ___)

P' (___, ___)

My slide: _____

b)



P (___, ___)

P' (___, ___)

My slide: _____

8. Draw a shape on a coordinate grid. Slide the shape and draw a translation arrow between a vertex and a corresponding vertex of the image. Describe how far the shape moved (right/left and up/down).

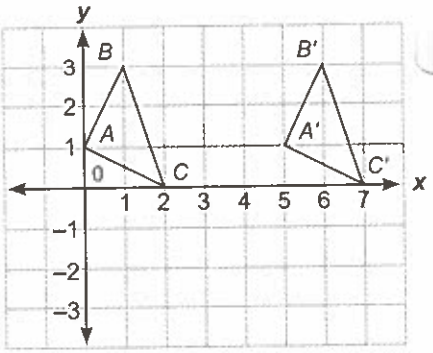
9. Draw each shape on a coordinate grid, translate it, and write the coordinates of its new vertices.

a) Square with vertices A (1, 1), B (1, 3), C (3, 3), D (3, 1) Translate 3 units right, 4 units up

b) Triangle with vertices A (3, 7), B (2, 5), C (5, 4) Translate 4 units right, 3 units down

G8-10 Translations — Advanced

INVESTIGATION ▶ How do coordinates change under translation?



A. Triangle *ABC* slid 5 units to the right. Write the coordinates of its vertices before and after the slide.

before: *A* (___, ___) *B* (___, ___) *C* (___, ___)

after: *A'* (___, ___) *B'* (___, ___) *C'* (___, ___)

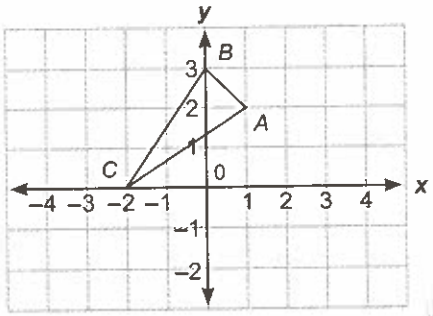
B. Which coordinate changed during the translation, the *x*-coordinate or the *y*-coordinate? _____

C. Look for a pattern: The ____ -coordinate increased by ____ .

D. Use the pattern in C. to predict the coordinates of these points after they slide 5 units right.

a) *D* (0, –2) ⟶ *D'* (5, –2) b) *E* (–1, –3) ⟶ *E'* (___, ___) c) *F* (–2, 2) ⟶ *F'* (___, ___)

E. Plot points *D*, *E*, *F* and *D'*, *E'*, *F'* on the grid above to check your prediction.



F. Slide triangle *ABC* 3 units right. Compare the coordinates of the vertices before and after the slide.
Complete the algebraic expression for the *x*-coordinate.

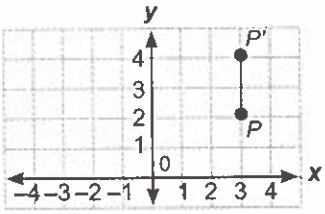
Point (*x*, *y*) slides 3 units right to point (*x +* ___, *y*)

Check the expression with three other points.

---

1. Slide the point *P* 2 units in the given direction. Write the coordinates of the new point. Which coordinate changed, and by how much?

a) 2 units up

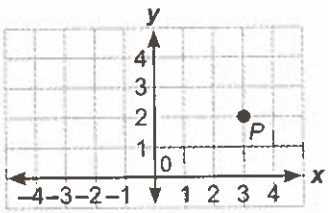


P (3, 2) *P'* (___, ___)

The *y* -coordinate

increased by 2

b) 2 units down

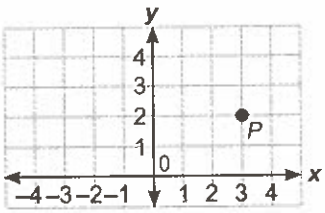


P (3, 2) *P'* (___, ___)

The ____-coordinate

_____

c) 2 units left



P (3, 2) *P'* (___, ___)

The ____-coordinate

_____

2. Point *Q* (*x*, *y*) slid to point *Q'*. Match the coordinates of *Q'* to the descriptions of translation.

D *Q'* (*x* + 4, *y*) ____ *Q'* (*x*, *y* – 4) ____ *Q'* (*x*, *y* + 4) ____ *Q'* (*x* – 4, *y*)

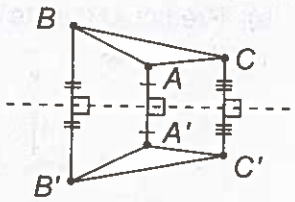
A. 4 units up B. 4 units down C. 4 units left D. 4 units right

3. A point (*x*, *y*) slides 3 units up and 2 units left. What are the coordinates of the new point? Draw a coordinate system and check your prediction for points (3, 3), (4, –2), (–1, 2), and (–3, –4).

G8-11 Reflections

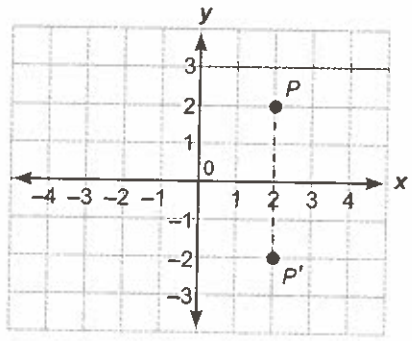
When a point is **reflected** in a mirror line, the point and the image of the point are the same distance from the mirror line.

The line between the point and the image is **perpendicular** to the mirror line.



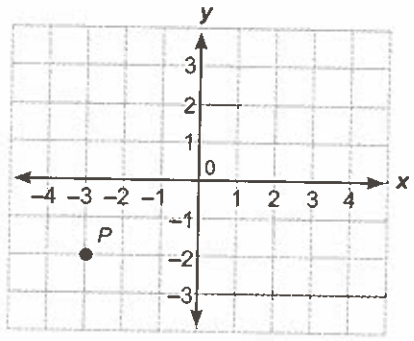
1. Reflect point P using the x-axis as a mirror line. Label the image point P'. Write the coordinates.

a)



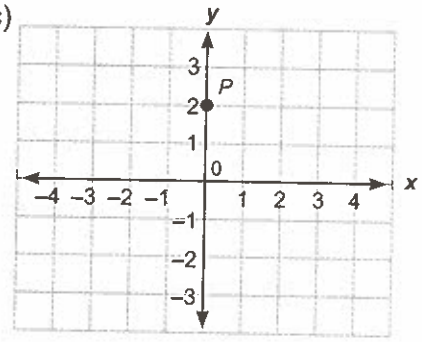
P (2 , 2) → P' (2 , −2)

b)



P (__, __) → P' (__, __)

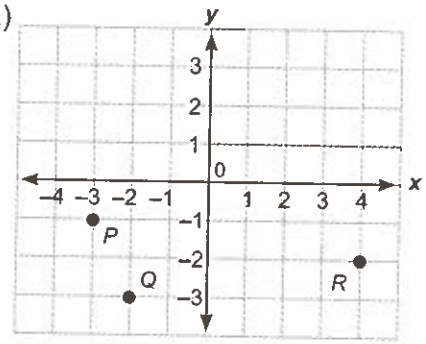
c)



P (__, __) → P' (__, __)

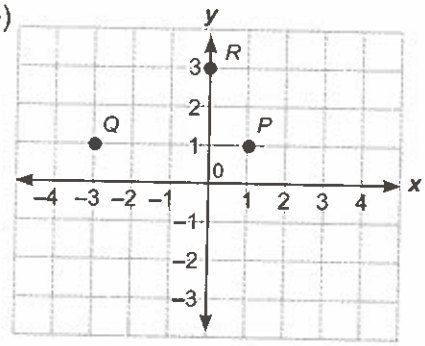
2. Reflect points P, Q, and R through the x-axis. Label the image points P', Q', and R' and write the coordinates. Hint: The image of a point on the x-axis through the x-axis is the point itself.

a)



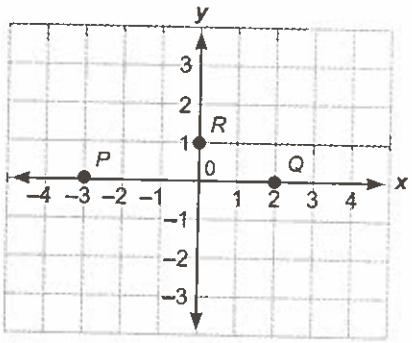
P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

b)



P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

c)



P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

3. Look at your answers in Question 2.

a) Which coordinate changed during the reflection through the x-axis?

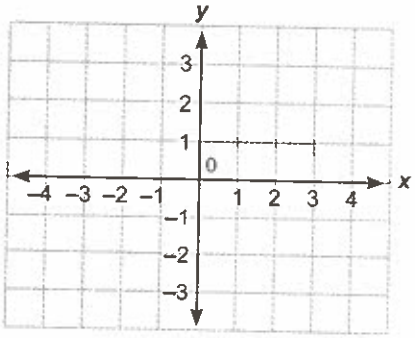
b) How did the coordinate in a) change?

c) Use your rule in b) to predict the coordinates of these points after reflection through the x-axis.

D (0, −3) → D' (0 , 3) E (−1, −4) → E' (__, __)

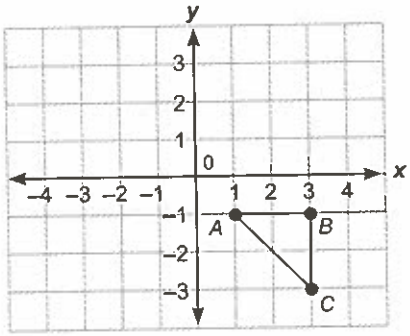
d) Plot points D, E and D', E' to check your prediction.

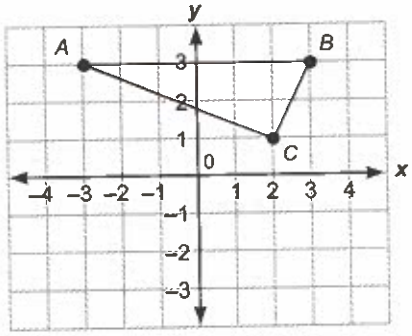
e) Use your rule in b) to explain why a point on the x-axis does not move when reflected through the x-axis.

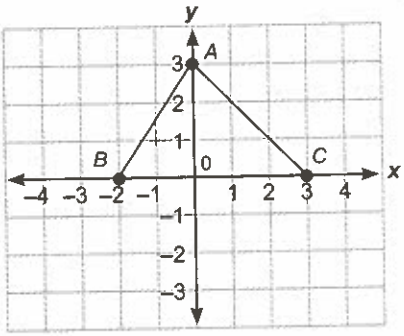


4. a) Write the coordinates of vertices *A*, *B*, and *C*.

b) Predict and write the coordinates of the vertices under a reflection through the *x*-axis.







A (__, __) → *A'* (__, __)
B (__, __) → *B'* (__, __)
C (__, __) → *C'* (__, __)

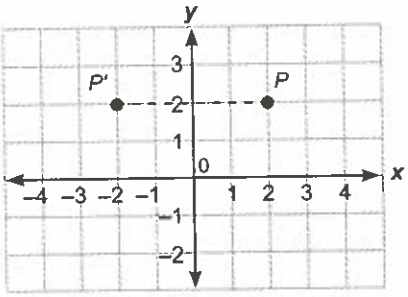
A (__, __) → *A'* (__, __)
B (__, __) → *B'* (__, __)
C (__, __) → *C'* (__, __)

A (__, __) → *A'* (__, __)
B (__, __) → *B'* (__, __)
C (__, __) → *C'* (__, __)

c) Reflect the figure by first reflecting the vertices through the *x*-axis, and check your answers from b).

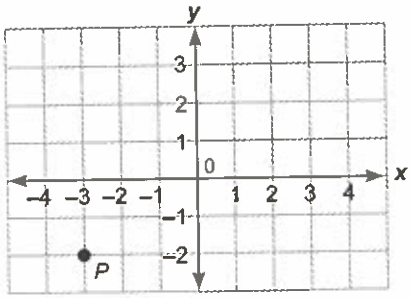
5. Reflect point *P* using the *y*-axis as a mirror line. Label the image point *P'*. Write the coordinates.

a)



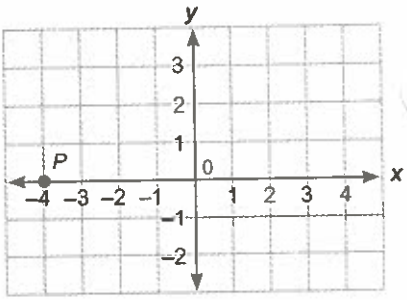
P (2, 2) → *P'* (−2, 2)

b)



P (__, __) → *P'* (__, __)

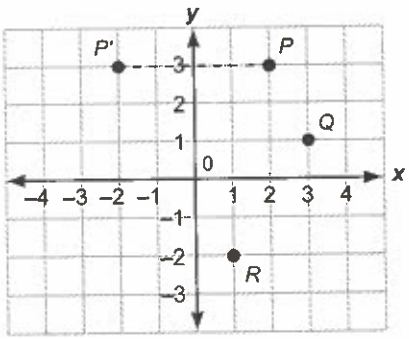
c)



P (__, __) → *P'* (__, __)

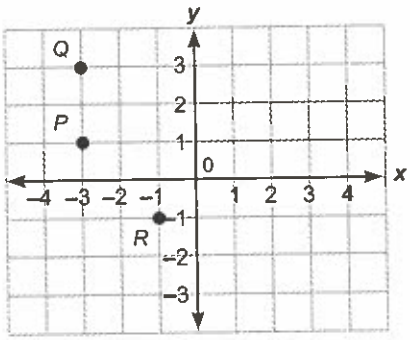
6. Reflect points *P*, *Q*, and *R* through the *y*-axis. Label the image points *P'*, *Q'*, and *R'* and write the coordinates. Hint: The image of a point on the *y*-axis through the *y*-axis is the point itself.

a)



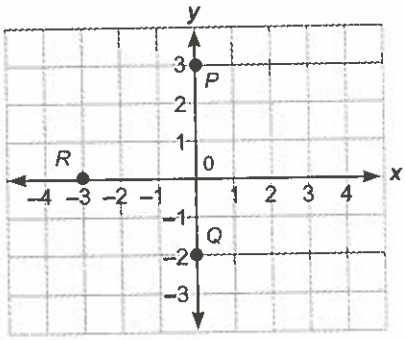
P (2, 3) → *P'* (−2, 3)
Q (__, __) → *Q'* (__, __)
R (__, __) → *R'* (__, __)

b)



P (__, __) → *P'* (__, __)
Q (__, __) → *Q'* (__, __)
R (__, __) → *R'* (__, __)

c)



P (__, __) → *P'* (__, __)
Q (__, __) → *Q'* (__, __)
R (__, __) → *R'* (__, __)

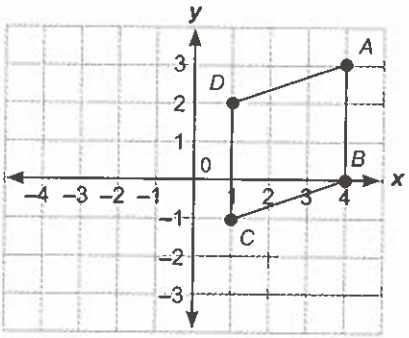
7. Look at your answers in Question 6.

a) Which coordinate changed during the reflection through the *y*-axis? _____

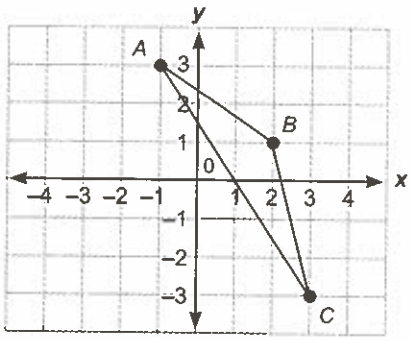
b) How did that coordinate change during the reflection through the *y*-axis? _____

8. a) Find the coordinates of the vertices of each shape.

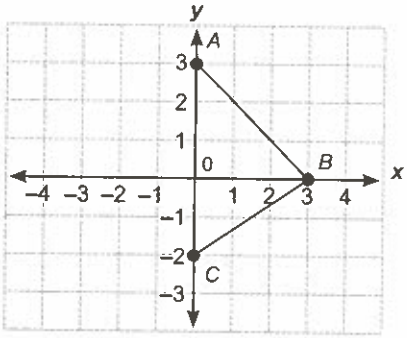
b) Predict and write the coordinates of the vertices under a reflection through the *y*-axis.



A (___, ___) → A' (___, ___)
B (___, ___) → B' (___, ___)
C (___, ___) → C' (___, ___)
D (___, ___) → D' (___, ___)



A (___, ___) → A' (___, ___)
B (___, ___) → B' (___, ___)
C (___, ___) → C' (___, ___)

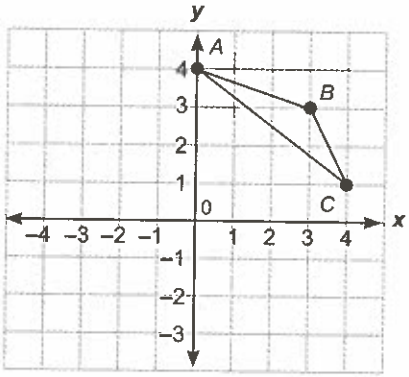


A (___, ___) → A' (___, ___)
B (___, ___) → B' (___, ___)
C (___, ___) → C' (___, ___)

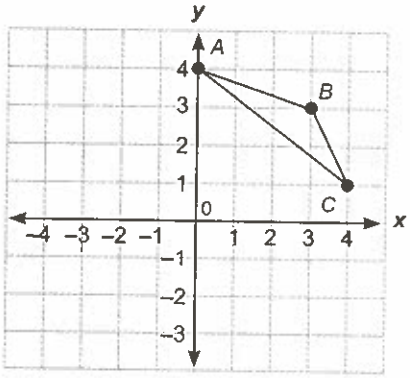
c) Reflect the figure by first reflecting the vertices through the *y*-axis, and check your answers from b).

Two figures are **congruent** if they are the same shape and size.

9. a) Reflect the triangle through the *x*-axis.



b) Slide the triangle 5 units down.



c) Use parts a) and b) to fill in the table with True or False.

	Reflection	Translation
The original figure and the image are congruent.	*True*	
The original figure and the image face in the same direction.		
The original figure and the image face in opposite directions.		

10. The pairs of triangles below are related by a reflection through the x-axis, a reflection through the y-axis, or a translation. Without plotting the triangles, say which transformation was used. Then plot the triangles to check your answers.

a) ΔABC : A (3, 1), B (3, 4), C (5, 2)

$\Delta A'B'C'$: A' (−3, 1), B' (−3, 4), C' (−5, 2)

$\Delta A'B'C'$ was obtained from ΔABC by

_____

b) ΔDEF : D (3, −1), E (3, −4), F (5, −2)

$\Delta D'E'F'$: D' (−3, −1), E' (−3, −4), F' (−1, −2)

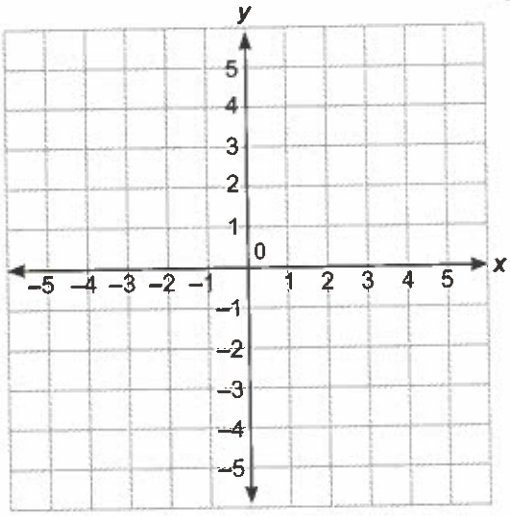
$\Delta D'E'F'$ was obtained from ΔDEF by

_____

c) ΔABC from a) and ΔDEF from b).

ΔABC was obtained from ΔDEF by

_____



11. a) Reflect ΔABC through the x-axis.

Label the image $\Delta A'B'C'$.

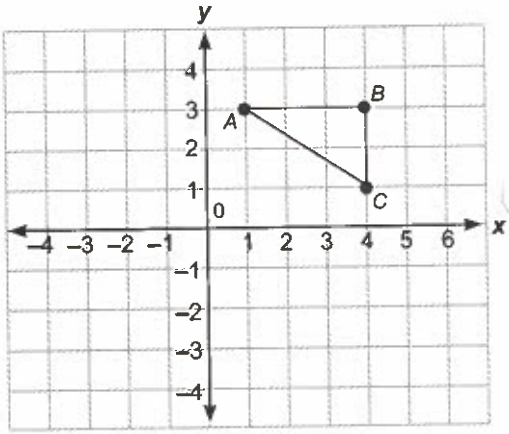
b) Reflect $\Delta A'B'C'$ through the y-axis.

Label the image $\Delta A''B''C''$.

c) Ying thinks that $\Delta A''B''C''$ is obtained from ΔABC by a translation. Is she correct? Explain why or why not.

_____

_____



12. a) Reflect $ABCD$ first through the x-axis, then through the y-axis.

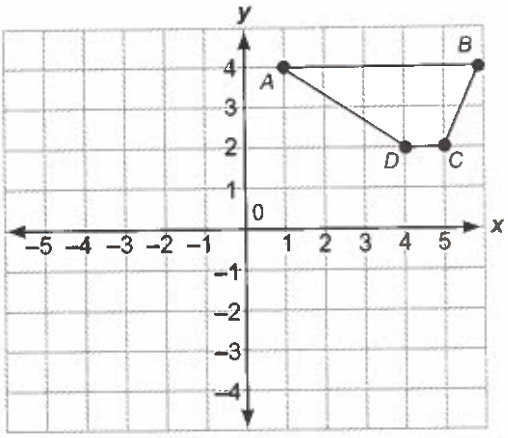
b) Reflect $ABCD$ first through the y-axis, then through the x-axis.

c) Did you get the same answer? _____

Explain why this happens using the rules for the change of coordinates for reflections that you developed in Questions 3 and 7.

_____

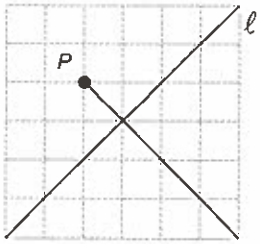
_____



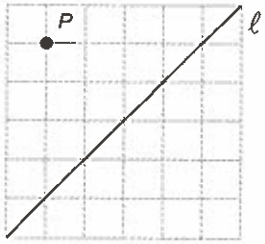
G8-12 Reflections — Advanced

1. Draw a line through P perpendicular to line ℓ. The first one is done for you.

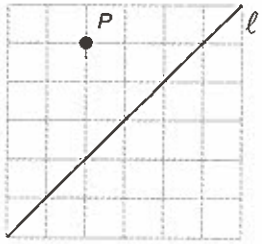
a)



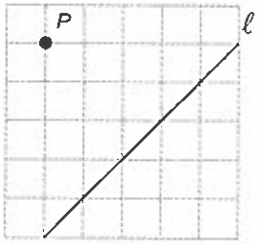
b)



c)



d)



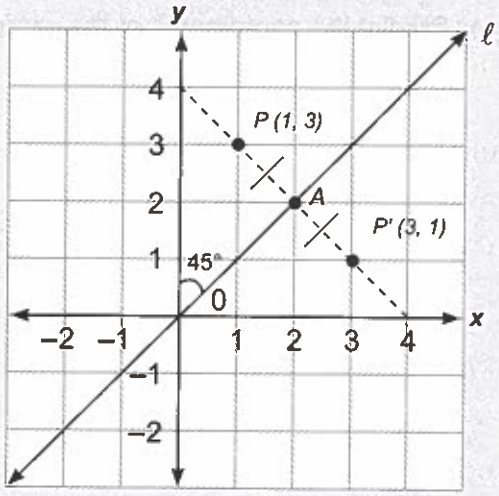
Line ℓ passes through the points
(1, 1), (2, 2), (3, 3), … and (–1, –1), (–2, –2), (–3, –3), ….
It passes through the first and third quadrants.

Example: Reflect point P (1, 3) through mirror line ℓ.

Step 1: Draw a dotted line through point P perpendicular to line ℓ.
Find the point where the dotted line meets ℓ and label it A.

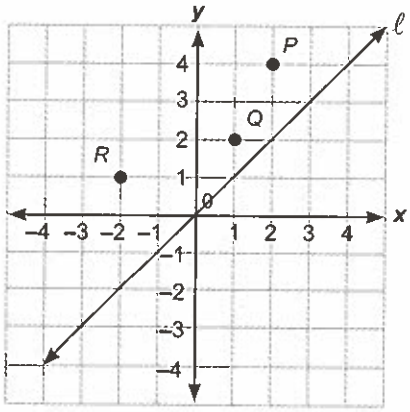
Step 2: Locate a point P' on the dotted line such that P' is the same distance from point A as P: $PA = P'A$.

P' is the image of P after reflection through line ℓ.



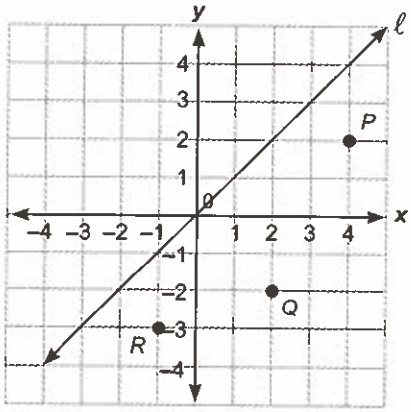
2. Reflect points P, Q, and R through mirror line ℓ. Label the image points P', Q', and R'.

a)



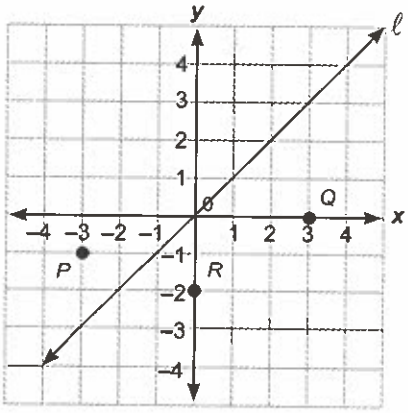
P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

b)



P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

c)



P (__, __) → P' (__, __)
Q (__, __) → Q' (__, __)
R (__, __) → R' (__, __)

3. Look at your answers in Question 2.

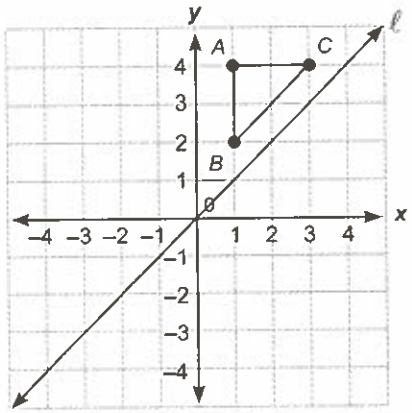
a) Describe how the coordinates of a point change during a reflection through line ℓ.

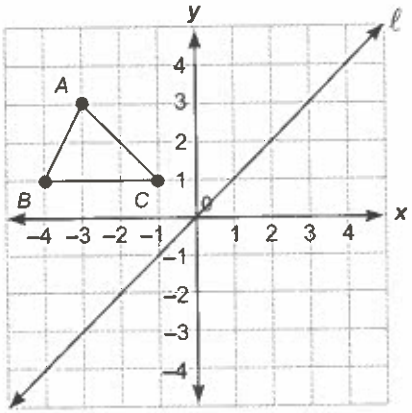
b) Use the rule you found in part a) to explain why a point on the line ℓ does not change when reflected through the line ℓ.

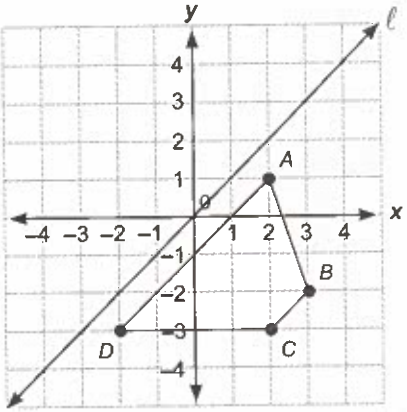
c) Use the rule from part a) to explain why a point in the second quadrant is reflected into the fourth quadrant. Why do the points in the first and third quadrant stay in the first and third quadrant?

4. a) Find the coordinates of the vertices of each shape.

b) Predict the coordinates of the vertices under a reflection through line ℓ.





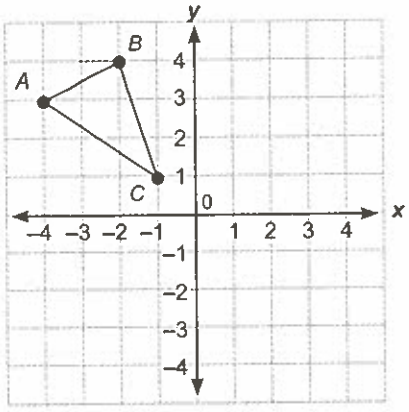


A (__, __) → A' (__, __)
B (__, __) → B' (__, __)
C (__, __) → C' (__, __)

A (__, __) → A' (__, __)
B (__, __) → B' (__, __)
C (__, __) → C' (__, __)

A (__, __) → A' (__, __)
B (__, __) → B' (__, __)
C (__, __) → C' (__, __)
D (__, __) → D' (__, __)

c) Reflect the figure by first reflecting the vertices through line ℓ, and check your answers from b).



BONUS ▶

a) Reflect ΔABC through the x-axis. Label the image $\Delta A'B'C'$.

b) Reflect $\Delta A'B'C'$ through the y-axis. Label the image $\Delta A''B''C''$.

c) Ling thinks that $\Delta A''B''C''$ is obtained from ΔABC by a reflection through line ℓ. Is she correct? Explain why or why not using the rule from Question 3 a).

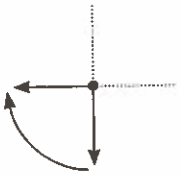
_____

_____

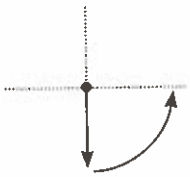
G8-13 Rotations

1. Draw the arrow after each turn. Start by drawing an arc to show where the final arrow should be.

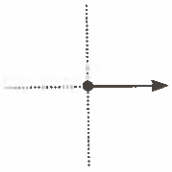
a) 90° clockwise



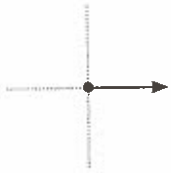
b) 90° counter-clockwise



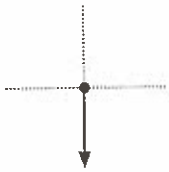
c) 90° clockwise



d) 90° counter-clockwise

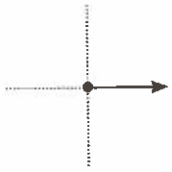


e) 180° clockwise



f) 180° counter-clockwise

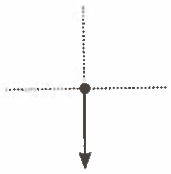
g) 180° clockwise



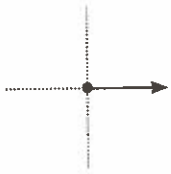
h) 180° counter-clockwise

i) 270° clockwise

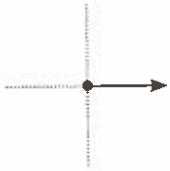
j) 270° counter-clockwise



k) 270° counter-clockwise



l) 270° clockwise

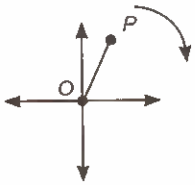


2. Match each rotation in the left column to a rotation in the right column that produces the same result. Hint: Use your answers from Question 1.

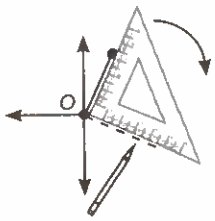
90° clockwise	90° counter-clockwise
180° clockwise	180° counter-clockwise
270° clockwise	270° counter-clockwise

Example: Rotate point P 90° clockwise about the origin using a set square.

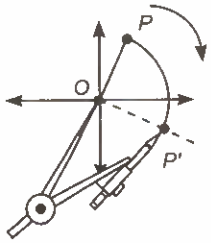
Step 1: Join P and the origin. Mark the direction of rotation.



Step 2: Place a set square and draw a line that makes a 90° angle with OP in the given direction.

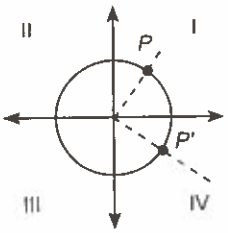


Step 3: Mark P' on your line so that $OP = OP'$. Use a ruler on the set square or a compass.



3. Using a set square and a circle instead of a compass, rotate P 90° clockwise. Label the image P'. Which quadrants are P and P' in?

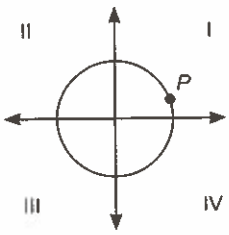
a)



P: Quadrant I

P': Quadrant IV

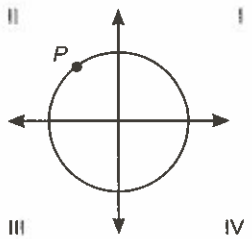
b)



P: Quadrant ___

P': Quadrant ___

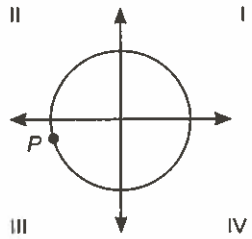
c)



P: Quadrant _____

P': Quadrant _____

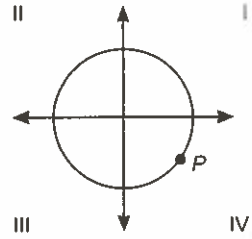
d)



P: Quadrant _____

P': Quadrant _____

e)



P: Quadrant _____

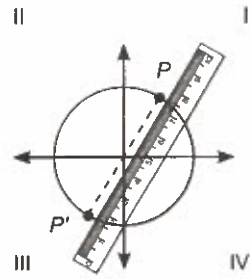
P': Quadrant _____

4. Using a set square (or ruler) and a circle, rotate P the given amount clockwise (CW) or counter-clockwise (CCW). Label the image P'. Which quadrants are P and P' in?

A **straight** angle measures 180°.

You can use a ruler to rotate a point around the origin 180° (clockwise or counter-clockwise).

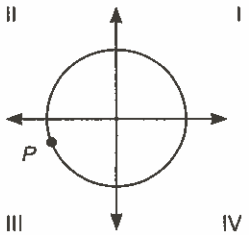
a) 180° CW



P: Quadrant _____

P': Quadrant _____

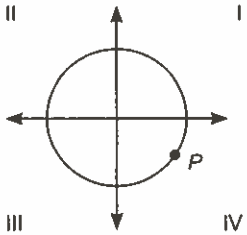
b) 180° CW



P: Quadrant _____

P': Quadrant _____

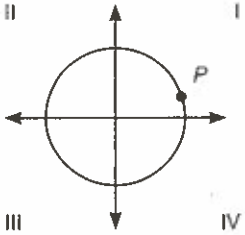
c) 180° CCW



P: Quadrant _____

P': Quadrant _____

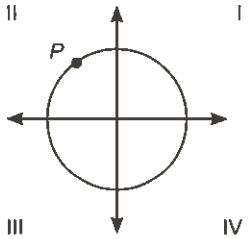
d) 90° CCW



P: Quadrant _____

P': Quadrant _____

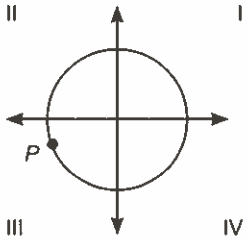
e) 180° CCW



P: Quadrant _____

P': Quadrant _____

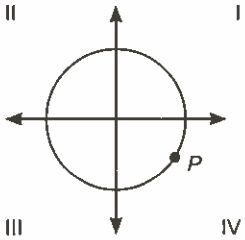
f) 90° CCW



P: Quadrant _____

P': Quadrant _____

g) 270° CW



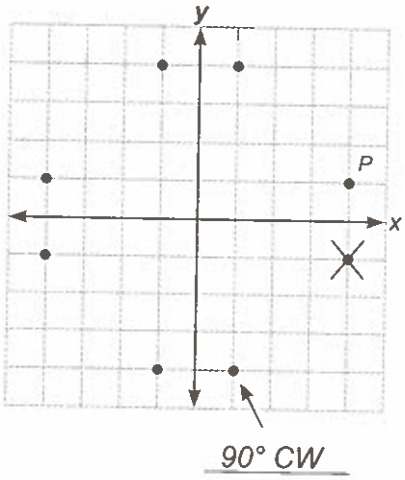
P: Quadrant _____

P': Quadrant _____

5. a) Cross out the points in the grid that cannot be obtained from P using a clockwise or counter-clockwise rotation around the origin by 90° or 180°. The first one is done for you.

b) Write the amount of rotation needed to obtain the remaining points from P. The first one is done for you.

BONUS ▶ Choose one of the points you crossed out in Quadrant II or IV. Describe both a reflection and a translation that would take P to this point.

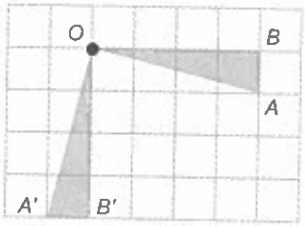


6. Congruent shapes have equal corresponding sides and equal corresponding angles. Triangles ΔOAB and $\Delta OA'B'$ are congruent.

a) Which sides and angles in $\Delta OA'B'$ are equal to these sides and angles in ΔOAB?

$OA =$ _____ $OB =$ _____ $AB =$ _____

$\angle AOB =$ _____ $\angle ABO =$ _____ $\angle OAB =$ _____



b) What is the degree measure of $\angle BOB'$? $\angle BOB' =$ _____

c) What is the degree measure of $\angle AOA'$? $\angle AOA' =$ _____. Explain.

_____

_____

d) Which transformation takes ΔOAB to $\Delta OA'B'$? _____

7. Point P (4, 1) is obtained by translating the origin 4 units right (horizontally) and 1 unit up (vertically).

Plot these points by translating the origin the given distances.

a) Point A: Horizontal translation: 2 units left

Vertical translation: 1 unit up

A (-2, 1) Quadrant II

b) Point B: Horizontal translation: 3 units right

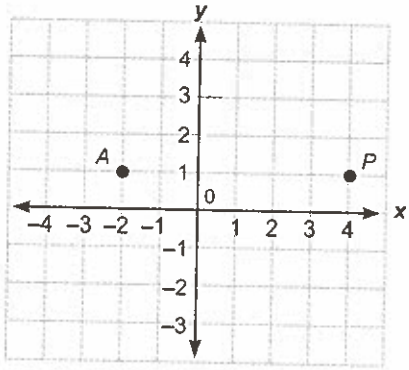
Vertical translation: 2 units down

B (___, ___) Quadrant _____

c) Point C: Horizontal translation: 2 units left

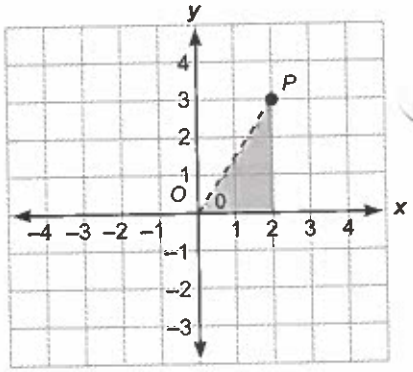
Vertical translation: 4 units down

C (___, ___) Quadrant _____



8. Choose one point in each quadrant of a coordinate grid. Describe the translation of the origin that is used to obtain each of the points.

9. Lina wants to rotate point P (2, 3) 90° clockwise around the origin.



a) Which quadrant is P in? _____

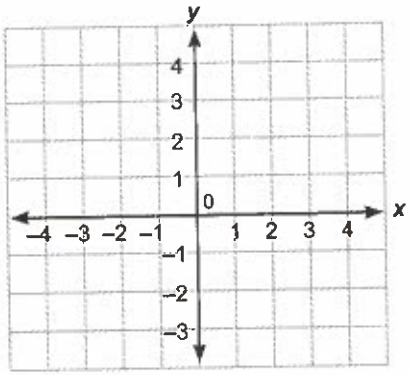
b) Which quadrant will the image P' be in? _____

c) Lina shades a right triangle with hypotenuse OP. Rotate her triangle 90° clockwise around the origin. Label the image P'.

d) The triangle before rotation has horizontal length 2 and vertical length 3.

The image triangle has horizontal length ___ and vertical length _____. P' has coordinates (___, ___).

10. Plot the point Q (–4, –2) on the coordinate grid.



a) Use Lina's method to rotate point Q 90° clockwise around the origin. Label the image Q'.

Q is in quadrant _____. Q' is in quadrant _____.

The triangle before rotation has horizontal length ___ and vertical length _____.

The image triangle has horizontal length ___ and vertical length _____. So Q' has coordinates (___, ___).

b) Use Lina's method to rotate point Q (–4, –2) 90° counter-clockwise around the origin. Label the image Q''.

Q'' is in quadrant _____.

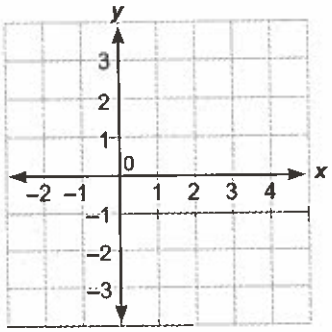
The triangle before rotation has horizontal length _____ and vertical length _____.

The image triangle has horizontal length _____ and vertical length _____. So Q'' has coordinates (___, ___).

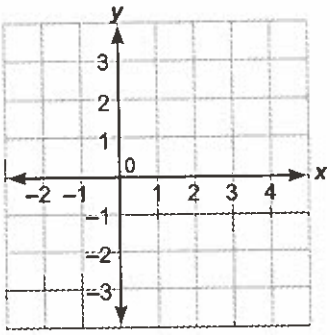
11. Compare the horizontal and the vertical lengths of the image triangles in Question 10. Are they equal? Are points Q' and Q'' the same? Explain.

12. Locate point P (3, 2) in the Cartesian plane. Use Lina's method to rotate P around the origin.

a) 90° clockwise P' (___, ___)



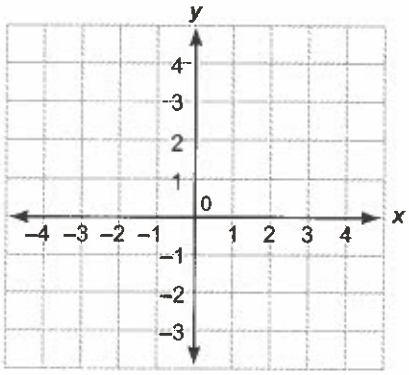
b) 90° counter-clockwise P'' (___, ___)



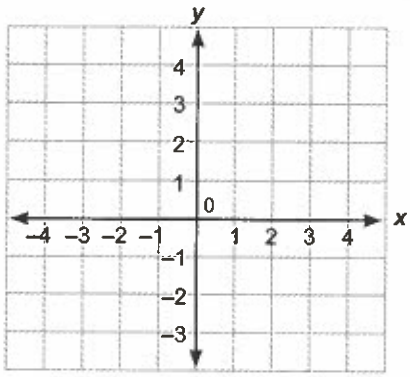
c) Which rotation would take point P' to point P''? _____

13. Locate point Q (–3, –2) in the Cartesian plane, then rotate Q around the origin.

a) 270° clockwise Q' (___, ___)

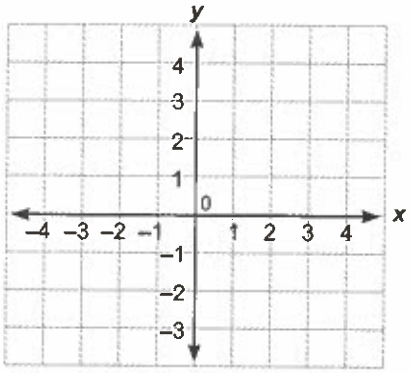


b) 270° counter-clockwise Q'' (___, ___)



14. Rotate each point P around the origin as given using Lina's method. Label the image point P'. Fill in the blanks.

a) P (–3, 1), 180° clockwise; P' (___, ___)

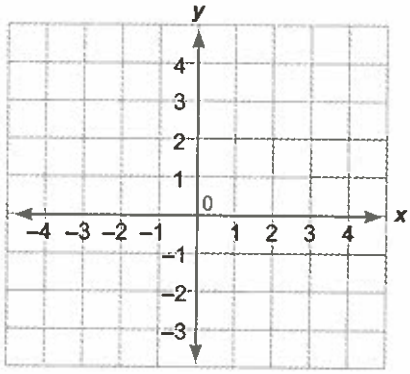


P is in quadrant ____. P' is in quadrant ____.

The triangle before rotation has horizontal length ____ and vertical length ____.

The image triangle has horizontal length ____ and vertical length ____.

b) P (4, –2), 180° counter-clockwise; P' (___, ___)



P is in quadrant ____. P' is in quadrant ____.

The triangle before rotation has horizontal length ____ and vertical length ____.

The image triangle has horizontal length ____ and vertical length ____.

15. a) Plot point P (4, 2) on the coordinate grid.

b) Rotate P 90° clockwise around the origin. P' (___, ___)

c) Rotate P' 180° clockwise around the origin. P'' (___, ___)

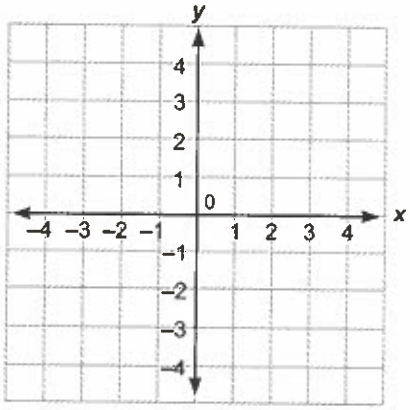
d) Point P'' can be obtained by rotating point P

____ clockwise around the origin.

e) Rotate P'' 270° clockwise around the origin. P''' (___, ___)

f) Point P''' can be obtained by rotating point P

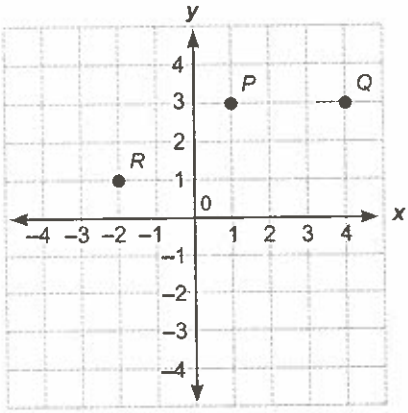
90° + 180° + 270° – 360° = ____ clockwise around the origin.
Explain where each number in this equation comes from.



G8-14 Transformations — Advanced

1. Rotate points P, Q, and R around the origin. Label the image points P', Q', and R'.

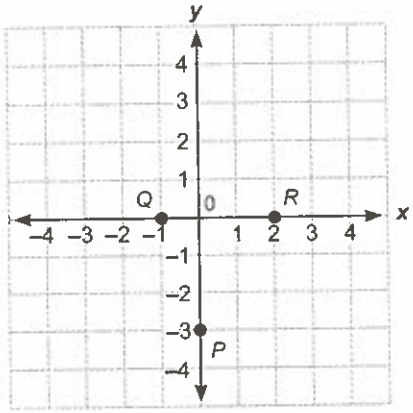
a) 90° counter-clockwise



P (___, ___), Q (___, ___), R (___, ___)

P' (___, ___), Q' (___, ___), R' (___, ___)

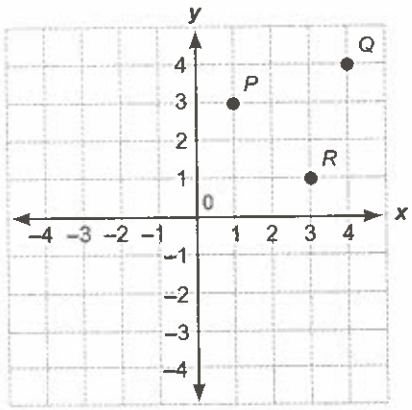
b) 270° counter-clockwise



P (___, ___), Q (___, ___), R (___, ___)

P' (___, ___), Q' (___, ___), R' (___, ___)

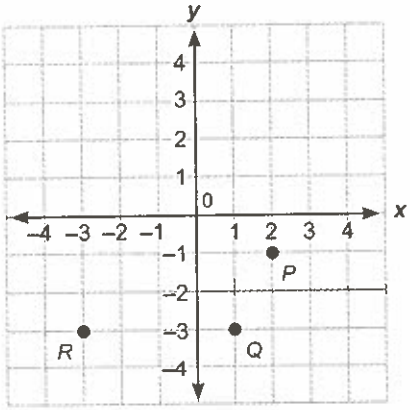
c) 180° counter-clockwise



P (___, ___), Q (___, ___), R (___, ___)

P' (___, ___), Q' (___, ___), R' (___, ___)

d) 270° counter-clockwise

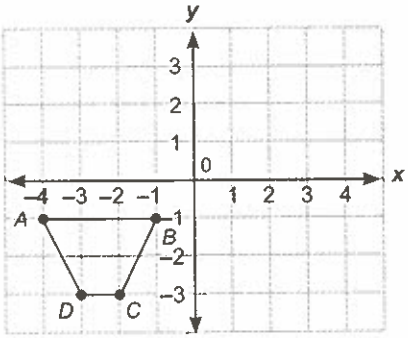


P (___, ___), Q (___, ___), R (___, ___)

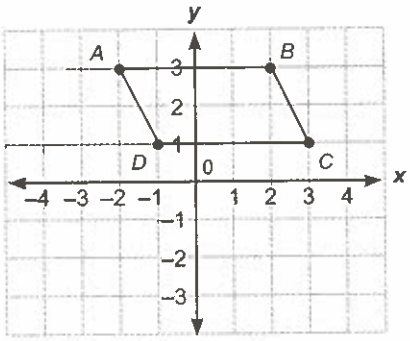
P' (___, ___), Q' (___, ___), R' (___, ___)

2. Rotate the figure around the origin by first rotating the vertices.

a) 90° clockwise

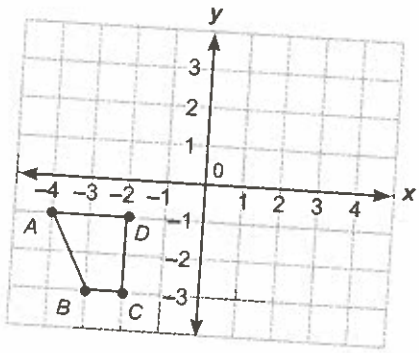


b) 90° clockwise

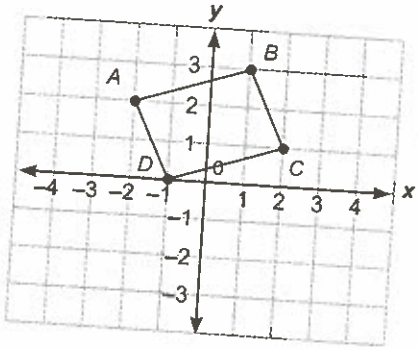


3. Rotate the figure around the origin by first rotating the vertices.

a) 180° clockwise



b) 180° clockwise



4. a) Write the coordinates of the points in 3 a) and b) in the table.

3 a)	original figure	*A* (__, __)	*B* (__, __)	*C* (__, __)	*D* (__, __)
	image figure	*A'* (__, __)	*B'* (__, __)	*C'* (__, __)	*D'* (__, __)
3 b)	original figure	*A* (__, __)	*B* (__, __)	*C* (__, __)	*D* (__, __)
	image figure	*A'* (__, __)	*B'* (__, __)	*C'* (__, __)	*D'* (__, __)

b) What do you notice about the coordinates of the images *A'B'C'D'* ?

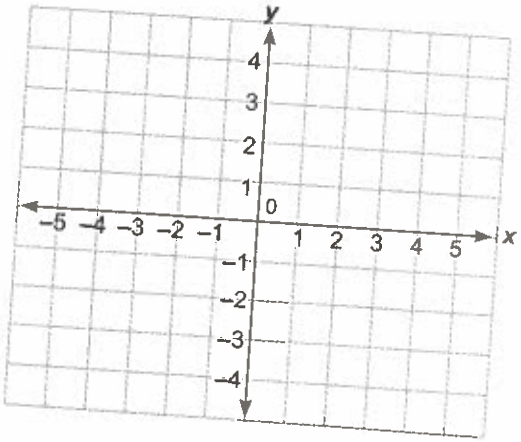
_____

_____

5. a) Plot these points in the Cartesian plane.

A (−1, −2), *B* (−1, −4),

C (−3, −4), *D* (−4, −2)



b) Predict the coordinates of the vertices of the image of *ABCD* under a 180° clockwise rotation.

A' (__, __) *B'* (__, __)

C' (__, __) *D'* (__, __)

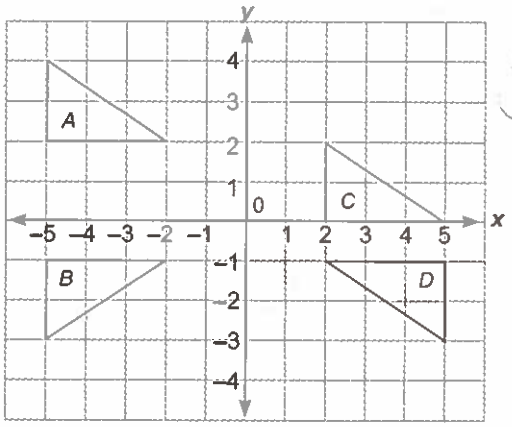
c) Rotate *ABCD* 180° clockwise to check your answer.

6. Which transformation changes triangle *A* into…

triangle *B*? _____

triangle *C*? _____

triangle *D*? _____



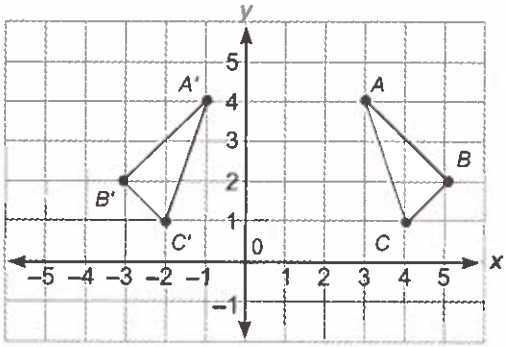
7. Draw a mirror line for the reflection.

A (___, ___) *B* (___, ___) *C* (___, ___)

A' (___, ___) *B'* (___, ___) *C'* (___, ___)

Which coordinate does not change in the reflection, the *x*-coordinate or the *y*-coordinate? Explain.

_____



8. Find two different points *F* and *G* so that Δ*DFE* and Δ*DGE* are congruent to Δ*ABC*. (Two figures are congruent if they have the same size and shape.)

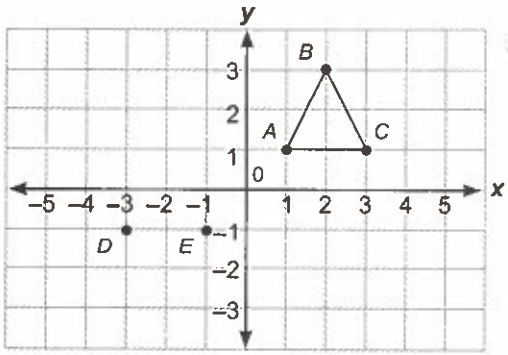
F (___, ___) *G* (___, ___)

Δ*DFE* was obtained from Δ*ABC* by

_____

Δ*DGE* was obtained from Δ*ABC* by

_____



9. Design your own transformations.

a) I rotate *ABCD* _____ degrees clockwise.

A' (___, ___) *B'* (___, ___)

C' (___, ___) *D'* (___, ___)

b) I translate *A'B'C'D'* ____ units ____ (up/down) and ____ units ____ (right/left).

A'' (___, ___) *B''* (___, ___)

C'' (___, ___) *D''* (___, ___)