

11.1

Determining Probabilities Using Tree Diagrams and Tables

Focus on...

After this lesson, you will be able to...

- determine the sample space of a probability experiment with two independent events
- represent the sample space in the form of a tree diagram or table
- express the probability of an event as a fraction, a decimal, and a percent



At the end of a unit on probability, Ms. Pascal decided to allow her students to determine what kind of test the class would write. All the students' names were put into a hat. Owen was chosen to spin a spinner divided into three equal regions to determine the kind of test: multiple choice (MC), short answer (SA), or a combination (MC & SA). Ava was chosen to roll a four-sided die to determine the number of questions on the test: 5, 10, 15, or 20.

Ms. Pascal explained that spinning the spinner and rolling the die are **independent events**. How does she know that these events are independent?

independent events

- results for which the outcome of one event has no effect on the outcome of another event

Explore the Math

How can you use the outcomes of an experiment to determine probabilities?

1. Show how you could represent the possible outcomes of this experiment.
2. What is the **probability** that the test will have multiple-choice questions only? How did you determine your answer?

Materials

- ruler

probability

- the likelihood or chance of an event occurring

3. What is the probability that the test will consist of ten questions? Explain your reasoning.
4. List the **sample space** for this experiment.

Reflect on Your Findings

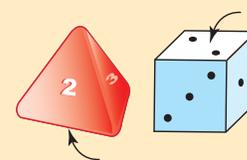
5. Show your answers to parts b), c), and d) as a fraction, a percent, and a decimal.
 - a) How many different tests are possible for the students in Ms. Pascal's class?
 - b) What is the probability that the students will write a combined multiple-choice/short-answer test with 20 questions? Show how you arrived at your answer.
 - c) What is the probability that students will write a multiple-choice test with at least ten questions?
 - d) What is the probability that the students will not write a short-answer test with 15 questions? Explain how you found your answer.

sample space

- all possible outcomes of a probability experiment

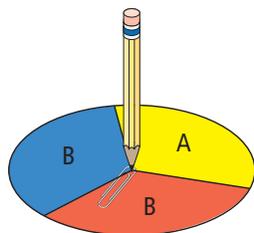
Did You Know?

When you roll a four-sided die, you read the number that is on the bottom. When you roll a six-sided die, you read the number on top.



Example 1: Determine Probabilities From a Tree Diagram

A spinner is divided into three equal regions as shown. The spinner is spun twice. For each probability you determine, express the answer as a fraction, a decimal, and a percent.



- a) What is the probability of spinning A on the first spin?
- b) Draw a tree diagram to represent the sample space for both spins.
- c) What is the probability of spinning A followed by B: $P(A \text{ then } B)$?
- d) What is the probability of getting the same letter on both spins: $P(A, A)$ or $P(B, B)$?

favourable outcome

- a successful result in a probability experiment

Solution

- a) The spinner has three equal regions: A, B, and B. There is only one favourable outcome, A, out of the three regions.

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(A) = \frac{1}{3} \\ = 0.\bar{3}$$

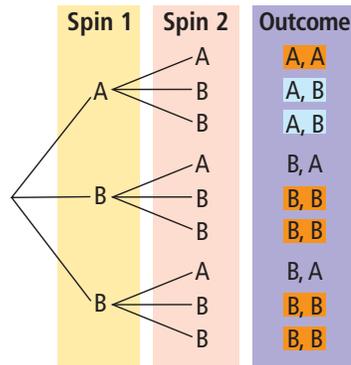


The probability of spinning an A is $\frac{1}{3}$, $0.\bar{3}$, or $33.\bar{3}\%$.

Strategies

Draw a Diagram

- b) The following tree diagram displays all possible outcomes.



- c) The tree diagram shows nine possible outcomes. There are two favourable outcomes (shaded blue).

$$\text{Probability} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$P(\text{A then B}) = \frac{2}{9} \\ = 0.\bar{2}$$



The probability of spinning A on the first spin and B on the second spin is $\frac{2}{9}$, $0.\bar{2}$, or $22.\bar{2}\%$.

- d) The favourable outcomes (shaded orange) in the tree diagram are (A, A), (B, B), (B, B), (B, B), (B, B). The probability that the same letter will appear on both spins is $\frac{5}{9}$, $0.\bar{5}$, or $55.\bar{5}\%$.

Since $\frac{2}{10}$ is 20%, **M E** the answer should be slightly greater than 20%.

Show You Know

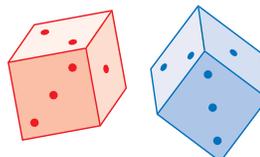
Ellen flips a coin and rolls a four-sided die numbered 1, 2, 3, and 4.



- What is the sample space? Use a tree diagram to show how you got your answer.
- What is $P(H, 4)$?

Example 2: Determine Probabilities From a Table

Two standard six-sided dice are rolled. One die is blue and the other is red. For each probability you determine, express the answer as a fraction, a decimal, and a percent.



- Create a table to represent the sample space.
- What is the probability of rolling a sum greater than ten?
- What is the probability that the number on the red die is one larger than the number on the blue die?
- What is the probability that the sum of the two numbers is less than 11?

Solution

- The following table represents the sample space. The numbers from the red die are shown in red and the numbers from the blue die are shown in blue.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Strategies

Make a Table

- b) The probability of rolling a sum greater than ten can be found by adding the two numbers in each cell of the table. There are three cells in the table with a sum greater than ten. So, there are three favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{sum} > 10) = \frac{3}{36}$$

$$= 0.08\bar{3}$$

$$\boxed{3} \div \boxed{36} = 0.08333333$$

The probability of a sum greater than ten is $\frac{3}{36}$, $0.08\bar{3}$, or $8.\bar{3}\%$.

- c) The probability that the number on the red die will be one larger than the number on the blue die can be found by counting favourable outcomes in the table.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$$P(\text{number on red die is one larger than number on blue die}) = \frac{5}{36}$$

$$= 0.13\bar{8}$$

$$\boxed{5} \div \boxed{36} = 0.13888889$$

The probability that the number on the red die is one larger than the number on the blue die is $\frac{5}{36}$, $0.13\bar{8}$, or $13.\bar{8}\%$.

- d) You can find the probability that the sum of the two numbers will be less than 11 by counting favourable outcomes.

		Blue Die					
		1	2	3	4	5	6
Red Die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Sometimes it is quicker to count the number of non-favourable outcomes and then subtract this number from the total number of possible outcomes. In this example, a non-favourable outcome is a sum greater than 10. There are three non-favourable outcomes.
 $36 - 3 = 33$
 favourable outcomes.

$$P(\text{sum} < 11) = \frac{33}{36} = 0.91\bar{6}$$

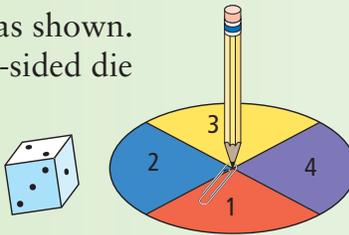
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33
÷
36
=
0.91666666

The probability that the sum of the two numbers is less than 11 is $\frac{33}{36}$, $0.91\bar{6}$, or $91.\bar{6}\%$.

Show You Know

A spinner is divided into four equal regions as shown. You spin this spinner and roll a standard six-sided die once each.

- Create a table to show the sample space.
- What is $P(4, 4)$?
- What is $P(\text{sum} > 5)$?

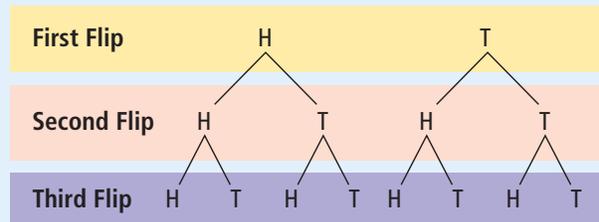


Key Ideas

- Probability = $\frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$
- The probability of both A and B occurring can be expressed as $P(A, B)$.
- The probability of event A occurring followed by event B can be expressed as $P(A \text{ then } B)$.
- You can use tree diagrams and tables to show the sample space for a probability experiment.
- Probabilities can be determined from tree diagrams and tables by direct counting of favourable outcomes and comparing the number of favourable outcomes with the total number of outcomes.

Communicate the Ideas

- John flips a coin and rolls a standard six-sided die.
 - What does the notation $P(H, 3)$ mean?
 - Explain how you could use a tree diagram to determine $P(H, 3)$.
- Monique missed class today. Explain to her how you could use this tree diagram to determine the probability of flipping a coin three times and getting exactly two heads and one tail.



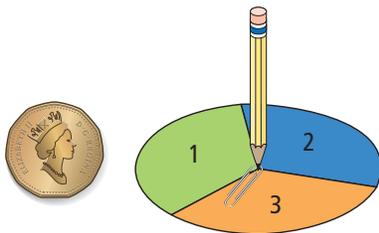
Check Your Understanding

Practise

Express all probabilities as a fraction, a decimal, and a percent.

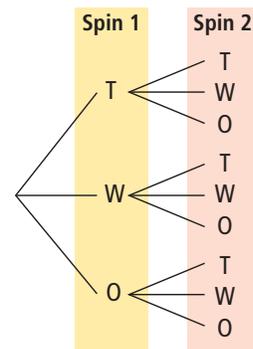
For help with #3 and #4, refer to Example 1 on pages 411–412.

- A spinner is divided into three equal regions as shown. Damien flips a coin and spins the spinner once.



- Draw a tree diagram to represent the sample space.
- List the sample space.
- What is the probability of $P(H, 2)$?

- The following tree diagram represents the sample space for a probability experiment.



- What is the sample space for this experiment?
- What is $P(T, W)$?
- What is the probability that both letters are identical?

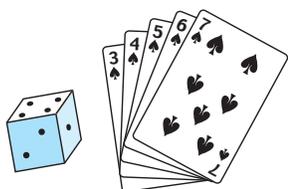
For help with #5 and #6, refer to Example 2 on pages 413–415.

5. Two four-sided dice are each rolled once. Each die is numbered 1, 2, 3, and 4.



- Create a table to represent the sample space.
- What is the probability that the sum is greater than five?
- What is the probability that the same number is the outcome on both dice?

6. Ali draws a card at random from the set of five cards pictured and rolls a standard six-sided die once.



- Create a table to show the sample space.
- What is the probability that the same number is the outcome on both the card and die?
- What is the probability that the sum of the two numbers is even?
- What is the probability that the number on the die is equal to or larger than the number on the card?

Apply

7. Lucy is jigging for fish through the ice. She has an equal chance of catching a whitefish, a trout, an arctic char, or losing the fish. If she pulls her hook out twice, what might she catch?
- Draw a table showing the results of Lucy's fishing.
 - What is $P(\text{whitefish, char})$ in either order?
 - What is $P(\text{char, char})$?
 - What is the probability she will catch nothing at all?

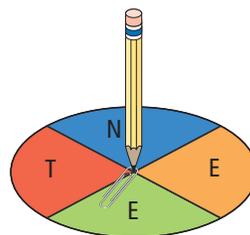
8. The sample space for the flip of a coin and a randomly picked card from five playing cards is (H, 6), (H, 7), (H, 8), (H, 9), (H, 10), (T, 6), (T, 7), (T, 8), (T, 9), and (T, 10).

- Draw a tree diagram to show the sample space.
- Construct a table to show the sample space.
- What is the probability that the result of this experiment includes an even-numbered card?

9. Two babies were born today.

- Construct a table to show the possible genders for the two babies.
- What is the probability that there is one boy and one girl?
- What assumption did you make about the likelihood of a boy or girl being born?

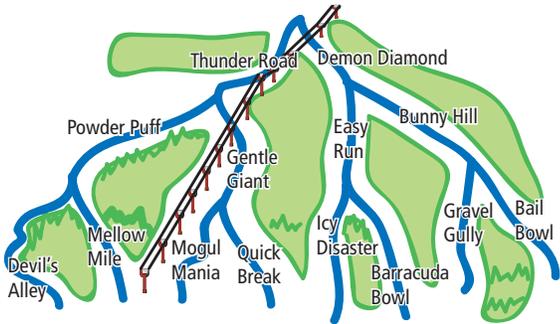
10. A spinner is divided into four equal regions. The spinner is spun twice.



- Create a table to show the sample space.
- What is the probability of spinning a T and then an E: $P(\text{T then E})$?
- What is $P(\text{E, E})$?
- What is $P(\text{same letter on both spins})$?

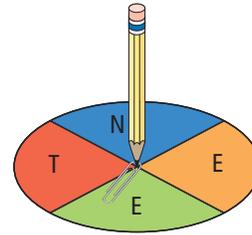
Extend

11. Nick and Manny are snowboarding in the Rockies. On one run down the mountain, they decide to flip a coin to choose which of two paths they will take at each of the three places where the ski runs branch. They will go down the left ski run if the coin is a head and the right ski run if the coin is a tail.



- What is the probability that they will take Thunder Road?
- What is the probability that Nick and Manny will finish on a run containing the name *Bowl*?
- What is the probability that they will take Thunder Road and Quick Break? Explain your answer.

12. A spinner is divided into four equal regions. The spinner is spun three times.



- Draw a tree diagram to show the sample space.
 - What is the probability of $P(E, E, E)$?
 - What is the probability of spinning three different letters in alphabetical order?
 - What is the probability that one letter appears exactly twice?
13. Alena rolls two standard six-sided dice.
- What is the probability that the difference between the two numbers is two?
 - What is the probability that the sum is a multiple of three?
 - What is the probability that the product is a multiple of four?

MATH LINK

The stick game uses four flat sticks. One side of each stick is bare and the other side is decorated. The four sticks are tossed in the air and allowed to fall to the ground. The score depends on the number of decorated sides that land facing up.

- Draw a tree diagram or create a table to show the possible outcomes.
- At the end of each branch or in each cell, record the total number of decorated sides showing.
- What is the probability of exactly three sticks landing decorated side up?



Did You Know?

Originally, rib bones from a buffalo or deer were used for the stick game.

11.2

Outcomes of Independent Events

Focus on...

After this lesson, you will be able to...

- determine the outcomes of two or more independent events
- verify the total number of possible outcomes using a different strategy



You make many choices every day. Ethan's decision is simple when he looks at today's cafeteria menu. He loves soup, cheeseburgers, and chocolate milk. But Sarah likes all the items listed on the menu. How many choices does she have? A lunch special consists of one appetizer, one main meal, and one drink. How many different lunch specials can you create for her to choose from?

Explore the Math

How do you determine the total number of possible outcomes?

1. Use a method of your choice to show Sarah's possible choices. You may wish to use abbreviations for the choices, such as CB for cheeseburger.
2. Compare your method of showing Sarah's choices to those of other classmates. What other methods were used? Which method is the most efficient? Justify your choice.
3. How many possible outcomes did Sarah have?

Literacy Link

The order is not important in a combination. For example, (juice, cookie) is the same combination as (cookie, juice).

4. Determine the number of possible outcomes for each combination given in the table.

Number of Choices for Item 1	Number of Choices for Item 2	Number of Choices for Item 3	Outcomes
2 types of ice-cream cones	3 flavours of ice cream	none	
5 shirts	4 pants	none	
5 models of sports cars	3 different colours	none	
4 models of computers	4 models of monitors	none	
3 models of computers	4 models of monitors	2 models of printers	

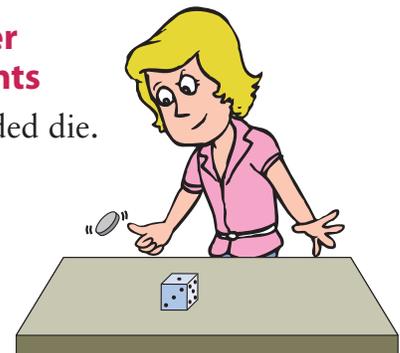
5. Study the numbers in each row of the table, looking for any patterns. How could you calculate the total number of outcomes using the number of choices for each item?

Reflect on Your Findings

6. a) Use your conclusion from #5 to calculate the number of different lunch specials available to Sarah. Explain your reasoning. Compare your result with your answer to #3.
- b) At another school's cafeteria, Martha has two choices of soup, four choices of main dish, two choices of dessert, and three choices of a beverage. Determine the number of possible lunch specials she can choose from. Show that you are correct by using another method.
- c) Suggest a rule for determining the total number of possible outcomes in a series of independent events without creating a tree diagram or table.

Example 1: Determine the Total Number of Outcomes From Two Events

Carrie flips a coin and rolls a standard six-sided die. How many possible outcomes are there?



Solution

Method 1: Create a Table

Coin Flip	Number on Die					
	1	2	3	4	5	6
H (head)	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
T (tail)	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

The table shows 12 possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes on die: 6

Number of possible outcomes on coin: 2

Total number of possible outcomes = 6×2
= 12

Multiply the number of possible outcomes for each event.

There are 12 possible outcomes.



Show You Know

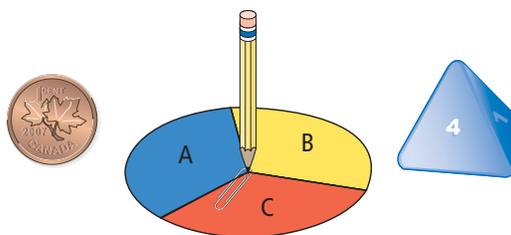
A café offers four types of sandwiches (egg salad, tuna, ham, or turkey) on one of three types of bread (white, rye, or whole wheat).

- Use a table to determine the number of sandwich combinations offered by the café.
- Check your answer using a different strategy.



Example 2: Determine the Total Number of Outcomes From Three or More Events

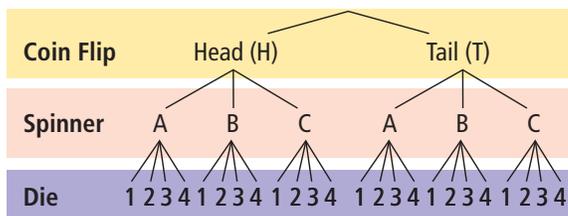
A coin is flipped, a spinner divided into three equal regions is spun, and a four-sided die numbered 1, 2, 3, and 4 is rolled.



- How many possible outcomes are there?
- Why could you not easily represent the sample space for this probability experiment with a table?

Solution

a) Method 1: Use a Tree Diagram



The tree diagram shows 24 possible outcomes.

Method 2: Use Multiplication

Number of possible outcomes for coin flip: 2

Number of possible outcomes for spinner: 3

Number of possible outcomes for die: 4

$$\begin{aligned} \text{Total number of possible outcomes} &= 2 \times 3 \times 4 \\ &= 24 \end{aligned}$$

There are 24 possible outcomes.

- b) A table is ideal for experiments that involve two events, because you can show one event in the columns and one event in the rows. You could not easily represent the sample space for this experiment in a table. For three events, you would need a three-dimensional table or more than one table in order to display all of the outcomes.

Show You Know

A café offers three types of sandwiches (cheese, chicken salad, or tuna) on one of the three types of bread (white, whole wheat, or spelt) with one of two choices of side orders (carrots or chips).

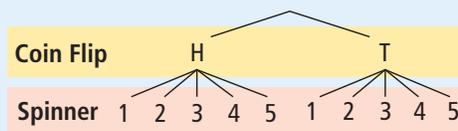
- How many possible combinations are offered by the café?
- Check your answer using a different strategy.

Key Ideas

- The total number of possible outcomes can be determined by counting outcomes shown in a table or tree diagram.

Coin Flip	Number on Spinner				
	1	2	3	4	5
H (head)	H, 1	H, 2	H, 3	H, 4	H, 5
T (tail)	T, 1	T, 2	T, 3	T, 4	T, 5

Total number of possible outcomes from the table: 10



Total number of possible outcomes from the tree diagram: 10

- The total number of possible outcomes can also be determined by multiplying the number of possible outcomes for each event.

Number of possible outcomes from coin flip: 2

Number of possible outcomes from spinner: 5

$$\begin{aligned} \text{Total number of possible outcomes} &= 2 \times 5 \\ &= 10 \end{aligned}$$

Communicate the Ideas

- Jasmine wrote a different number from one to ten on each of ten small pieces of paper and put them in a bag. She drew one number from the bag. At the same time, she flipped a coin. Using three different methods, show another student how to determine the total number of possible outcomes.
- Three flights travel from Lethbridge to Calgary each morning. Four flights go from Calgary to Edmonton in the afternoon. Show two methods for finding how many different ways you could fly from Lethbridge to Edmonton on a given day.
 - Which method is more efficient? Explain your thinking.

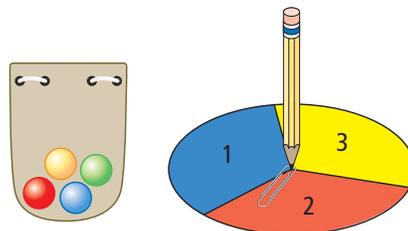


Check Your Understanding

Practise

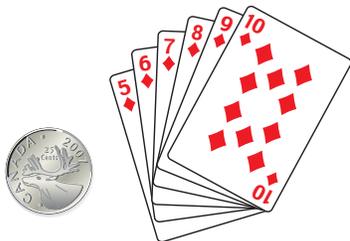
For help with #3 and #4, refer to Example 1 on pages 420–421.

- A bag contains four marbles: one green, one red, one blue, and one yellow. A spinner has three equal sections numbered 1, 2, and 3. A marble is randomly chosen from the bag and the spinner is spun.



- Display the sample space in a table.
- How many possible outcomes does the table show?
- Check your answer to part b) using another strategy.

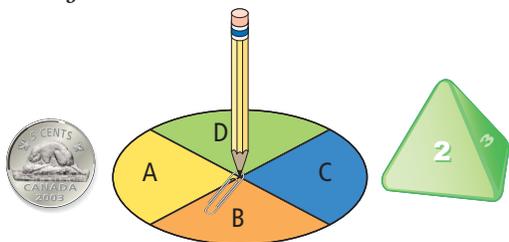
4. Wei flips a coin and randomly draws a card from the set of six cards shown.



- Use a method of your choice to determine the total number of possible outcomes.
- Verify your answer using a different strategy.

For help with #5 and #6, refer to Example 2 on pages 421–422.

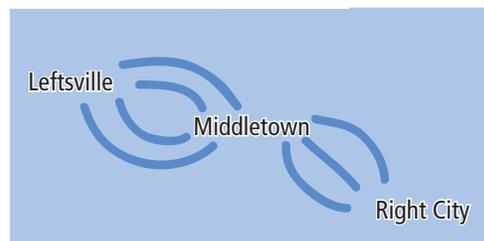
5. A coin is flipped, a six-sided die is rolled, and a marble is randomly selected from a bag containing one black, one yellow, and one red marble.
- Draw a tree diagram to organize the outcomes of these three events.
 - How many possible outcomes are there?
 - Use multiplication to verify the answer to part b).
6. Greta, Joe, and Jared do a probability experiment. Greta flips a coin, Joe spins a spinner divided into four equal regions, and Jared rolls a four-sided die.



- Use a tree diagram to organize the outcomes of these three events.
- How many possible outcomes are there for this probability experiment?
- Verify the answer to part b) by using multiplication.

Apply

- Tony has four different pairs of pants and six different shirts. How many shirt–pant combinations can he make?
- The map shows possible routes between three towns. How many possible routes could you take from Leftsville to Right City?



- The birthday menu at Blue Bird Restaurant gives you one choice from each category:
 Drink: four choices
 Meal: five choices
 Dessert: three choices
 How many different combinations are possible?
- Michaela has a nickel, a dime, and a loonie in her left jacket pocket. She has a penny and a quarter in her right pocket. She randomly picks one coin from each pocket.
 - How many combinations of coins could she get?
 - Use a second method to verify your answer to part a).
 - What is the largest sum possible for these two coins?
 - What is the smallest possible sum?
- Make up a question that would give the following number of possible outcomes:
 $2 \times 4 \times 5 = 40$

Extend

- 12.** An ice-cream store has 31 flavours of ice cream and three types of cones (waffle, sugar, and plain).
- Determine the number of possible single-scoop ice-cream cones.
 - How many two-scoop ice-cream cones are possible if waffle–chocolate–strawberry is considered different from waffle–strawberry–chocolate? Hint: You could have two scoops of the same flavour if you desired.
 - How many two-scoop cones are possible if both flavours of ice cream must be different? Explain your reasoning.
- 13.** A set meal consists of a choice of drink, main dish, and dessert. There are four different desserts, 36 possible meals in total, and more than one choice in each category. Determine the possible number of drink and main dish choices. Explain your reasoning.
- 14.** Alikut is planning to make a beaded yoke for her new parka. She has five colours of beads: red, blue, black, white, and yellow. She wants to use only six shapes of beads: square, circle, star, triangle, rectangle, and heart. Alikut plans to use only one colour of bead in each shape.
- How many colour–shape combinations could she use?
 - Use a second method to verify your answer to part a).
 - Suppose Alikut decides to use two colours of beads in each shape. How many colour–shape combinations could she use now?
- 15.** Determine the number of four-digit numbers that contain only the digits 1, 2, 3, and 4. A digit can be repeated. Two valid numbers are 1423 and 4442.
- 16.** How many car license plates can be made if the first three characters are letters and the last three characters are digits from 2 through 9 inclusive?

MATH LINK

In the stick game, each stick can land in one of two ways—decorated or plain side up.

- Use a different method than you used in section 11.1 to confirm the total number of possible outcomes for a game with four sticks.
- Sometimes the game uses different numbers of sticks. What is the total number of possible outcomes for three sticks? five sticks?
- If there are 128 possible outcomes, how many sticks are being used?

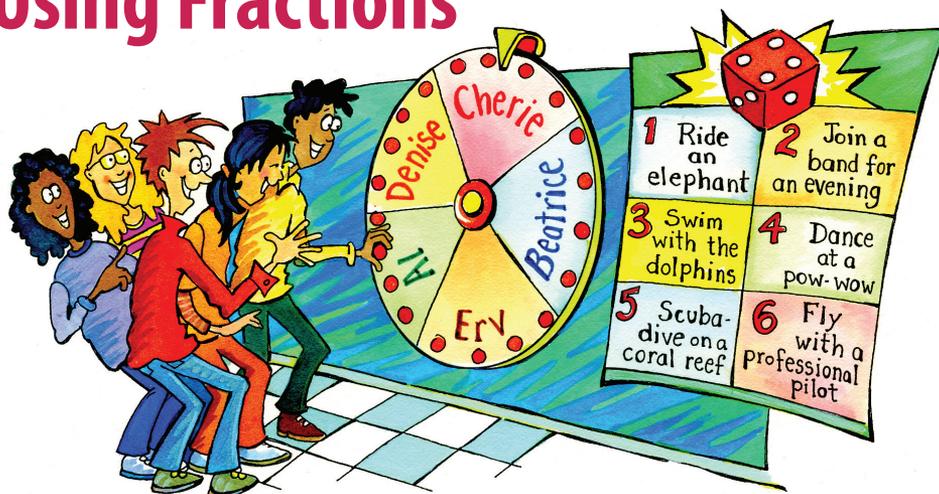
11.3

Determining Probabilities Using Fractions

Focus on...

After this lesson, you will be able to...

- solve probability problems
- verify your answers using a different method



Erv and his friend Al have been chosen as contestants in a new TV reality program called Wheel of Thrills. Five contestants start the game. A wheel is divided into five equal sectors and labelled with each person's name. The wheel is spun once to determine who will be the potential winner for the 30-minute show. Once a person is selected, a standard six-sided die is rolled to determine what kind of thrill he or she will experience.

Erv and Al both love to swim. How likely do you think it is that one of these boys will be chosen and will get to swim with dolphins or scuba-dive on a coral reef?

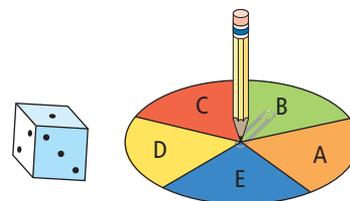
Explore the Math

Materials

- ruler
- red and yellow pencils

How can you determine probabilities using fractions?

1. a) Copy the table into your notebook.



		Number on Die					
		1	2	3	4	5	6
Name of Contestant	Al						
	Beatrice						
	Cherie						
	Denise						
	Erv						

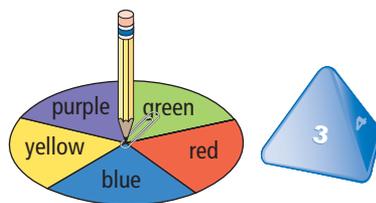
- b) Use a red pencil to shade the rows that correspond to the spinner landing on Al or Erv's name. What fraction of the five rows did you shade?
 - c) Use a yellow pencil to shade the columns that correspond to the die roll showing swimming with dolphins or scuba-diving on a coral reef. What fraction of the six columns did you shade?
2. a) What fraction of the total number of cells in the table are shaded both red and yellow? Do not write this fraction in lowest terms.
- b) How could you use the fractions from #1 to determine the fraction of the total number of cells that are shaded both red and yellow?
 - c) What probability does this fraction represent?

Reflect on Your Findings

3. a) How could you use multiplication to calculate the total number of possible outcomes for this experiment?
- b) How is the total number of possible outcomes related to your answer to #2c)?
 - c) How is the number of outcomes that thrill Al or Erv related to your answer to #2c)?
 - d) How can you use the probabilities of single events to determine the probability of two independent events happening?

Example 1: Calculating Probabilities Using a Table and Multiplication

Mackenzie spins a spinner divided into five equal regions and rolls a four-sided die once each.



- a) Construct a table to represent the sample space. How many possible outcomes are there?
- b) From the table, what is $P(\text{blue}, 2)$ expressed as a fraction?
- c) Use multiplication to determine $P(\text{blue}, 2)$.
- d) From the table, what is $P(\text{red or blue}, < 4)$ expressed as a fraction.
- e) Use the method from part c) to calculate $P(\text{red or blue}, < 4)$.

The $<$ symbol means *less than*. In part d), rolling less than a four means rolling a one, two, or three.

Strategies

Make a Table

Solution

a)

Spinner	Four-Sided Die			
	1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

Number of possible outcomes: 20

- b) Label the Blue row in blue. Shade the column labelled 2. Identify the part of the table that is both labelled in blue and shaded.

Spinner	Four-Sided Die			
	1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

The table shows one favourable outcome.

$$P(\text{blue, 2}) = \frac{1}{20}$$

- c) The probability of spinning blue is $\frac{1}{5}$. The probability of rolling a 2 is $\frac{1}{4}$.

$$P(\text{blue, 2}) = \frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$$

- d) Colour your table to determine the probability of landing on red or blue and rolling 1, 2, or 3.

Spinner	Four-Sided Die			
	1	2	3	4
Blue	blue, 1	blue, 2	blue, 3	blue, 4
Red	red, 1	red, 2	red, 3	red, 4
Green	green, 1	green, 2	green, 3	green, 4
Yellow	yellow, 1	yellow, 2	yellow, 3	yellow, 4
Purple	purple, 1	purple, 2	purple, 3	purple, 4

The table shows six favourable outcomes.

$$P(\text{red or blue, } < 4) = \frac{6}{20}$$

Multiply the probabilities of the single events to determine the probability of the two independent events happening.

e) The probability of spinning red or blue is $\frac{2}{5}$.

The probability of rolling a 1, 2, or 3 is $\frac{3}{4}$.

$$P(\text{red or blue, } < 4) = \frac{2}{5} \times \frac{3}{4} \\ = \frac{6}{20}$$

Example 2: Calculating Probabilities Using a Tree Diagram and Multiplication

Jason rolls a standard six-sided die and Rachel spins a spinner with three equal sections. What is the probability of rolling an even number and spinning a B? Verify your answer using another method.

Solution

For the die: $P(\text{even number}) = \frac{3}{6}$

For the spinner: $P(B) = \frac{1}{3}$

$$P(\text{even number, B}) = P(\text{even number}) \times P(B) \\ = \frac{3}{6} \times \frac{1}{3} \\ = \frac{3}{18}$$

Use a tree diagram to verify your answer.

The tree diagram shows that there are 18 possible outcomes and three favourable outcomes.

$$P(\text{even number, B}) = \frac{3}{18}$$

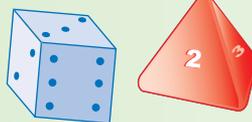
The tree diagram agrees with the result of the multiplication. The probability of rolling an even number and spinning a B is $\frac{3}{18}$ or $\frac{1}{6}$.



Die	Spinner	Outcome
1	A	1, A
	B	1, B
	C	1, C
2	A	2, A
	B	2, B
	C	2, C
3	A	3, A
	B	3, B
	C	3, C
4	A	4, A
	B	4, B
	C	4, C
5	A	5, A
	B	5, B
	C	5, C
6	A	6, A
	B	6, B
	C	6, C

Show You Know

A blue, standard six-sided die and a red, four-sided die numbered 1, 2, 3, and 4 are each rolled once. Determine the following probabilities, and then verify your calculations using a second method.



- a) $P(\text{blue} = 4, \text{red} = 4)$ b) $P(\text{blue} < 4, \text{red} < 4)$
 c) $P(\text{blue} = 4, \text{red} < 4)$

Literacy Link

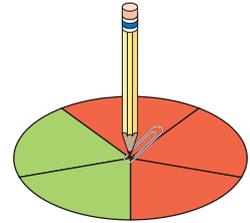
In a *simulation*, you model a real situation using an experiment.

Example 3: Simulations

Gina is planning the time needed to get to her soccer game. There are two traffic lights between her house and the soccer field. These lights are red (or yellow) 60% of the time. Gina wonders how likely it is that both lights will be red on her way to the game.

Model this situation by spinning a spinner divided into five equal regions twice. The table shows the results for ten trials.

Trial	Experimental Results		
	First Light (Green or Red)	Second Light (Red or Green)	Both Lights Red?
1	R	R	yes
2	G	G	no
3	R	G	no
4	G	R	no
5	R	R	yes
6	R	G	no
7	R	R	yes
8	G	G	no
9	G	R	no
10	G	G	no



Literacy Link

An *experimental probability* is the probability of an event occurring based on experimental results.

A *theoretical probability* is the calculated probability of an event occurring.

- What is the experimental probability that both lights are red?
- What is the theoretical probability that both lights are red?
- Compare the experimental probability with the theoretical probability. How could Gina improve the accuracy of the experimental probability?

Solution

- a) From the table, there are three favourable outcomes.

$$\begin{aligned}P(\text{both lights red}) &= \frac{3}{10} \\ &= 0.3\end{aligned}$$

The experimental probability that both lights are red is $\frac{3}{10}$, 0.3, or 30%.

- b) The probability that one traffic light is red is 60% or $\frac{3}{5}$.

$$\begin{aligned}P(\text{both lights red}) &= \frac{3}{5} \times \frac{3}{5} \\ &= \frac{9}{25} \\ &= 0.36\end{aligned}$$

The theoretical probability that both lights are red is $\frac{9}{25}$, 0.36, or 36%.

- c) The experimental probability is lower than the theoretical probability. If Gina performed more trials of the experiment, the two probabilities would likely be closer in value.

Show You Know

Andrew flips two coins to simulate the genders of the children in families with two children. He decides that heads indicates a girl and tails indicates a boy. The following chart shows his results for 100 simulations. Express your answers to parts a) and b) as a decimal and a percent.

Coin Outcomes	HH	HT	TH	TT
Child Outcomes	two girls	girl, boy	boy, girl	two boys
Number of Results	27	24	27	22

- a) What is Andrew's experimental probability of getting two boys?
 b) What is the theoretical probability of getting two boys? Use multiplication to determine your answer.
 b) Compare the experimental probability with the theoretical probability. Why are the two values different?

Key Ideas

- For probability experiments involving two or more independent events, the probability can be found by multiplying the probabilities of success for each single event.

A spinner with three equal regions labelled 1, 2, and 3 is spun and a coin is flipped. What is the probability of spinning a 2 and flipping tails?

Using Multiplication:

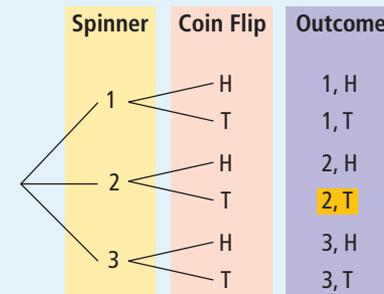
$$\begin{aligned}
 P(2, T) &= P(2) \times P(T) \\
 &= \frac{1}{3} \times \frac{1}{2} \\
 &= \frac{1}{6}
 \end{aligned}$$

Using a Table:

Spinner	Coin Flip	
	Head	Tail
1	1, H	1, T
2	2, H	2, T
3	3, H	3, T

$$P(2, T) = \frac{1}{6}$$

Using a Tree Diagram:

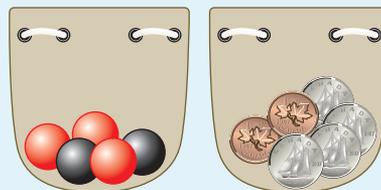


$$P(2, T) = \frac{1}{6}$$

- A simulation is an experiment that can be used to model a real situation. The results of a simulation are called experimental results.

Communicate the Ideas

1. A bag contains three red marbles and two black marbles. A second bag contains two pennies and four dimes. A marble and a coin are drawn from each bag at random. Explain to a classmate who missed the lesson how to calculate $P(\text{red, penny})$ using multiplication.



2. Catherine gives the following explanation for how to calculate $P(\text{black, dime})$. She says that there are two choices for marbles (red and black) and two choices for coins (pennies and dimes).

$$\begin{aligned} P(\text{black, dime}) &= \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{4} \end{aligned}$$

What mistake did Catherine make?

3. Explain the difference between experimental probability and theoretical probability.

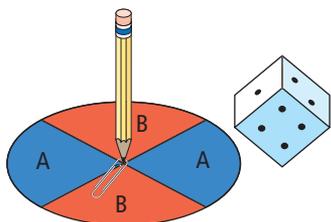
Check Your Understanding

Practise

Express all probabilities as fractions in lowest terms unless otherwise specified.

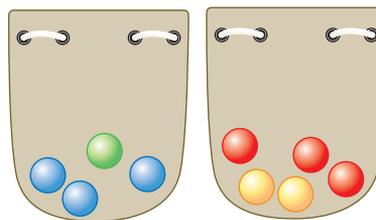
For help with #4 and #5, refer to Example 1 on pages 427–429.

4. Brittany spins a spinner divided into four equal regions and rolls a standard die once.



- a) Construct a table to organize the sample space.

- b) What is the probability of spinning an A and rolling a two?
 - c) Use a second method to determine $P(A, 2)$.
5. Joe takes one marble from the first bag and Ron takes one marble from the second bag.



- a) Use multiplication to calculate the total number of possible outcomes.
- b) What is the probability of $P(\text{blue, red})$? Show two different strategies for determining the answer.

For help with #6 and #7 refer to Example 2 on page 429.

6. A coin is flipped twice.
 - a) What is the probability that a head is flipped on the first flip, $P(H)$?
 - b) What is the probability that a head is flipped on both flips, $P(H, H)$?
 - c) Check both answers by using another method.

7. Levi rolls two dice, a six-sided one numbered from 1 to 6 and a four-sided one labelled A, B, C, and D.
 - a) Calculate $P(2, B)$.
 - b) Calculate $P(\text{even number, consonant})$.
 - c) Check your answers by using another method.

For help with #8 and #9 refer to Example 3 on pages 430–431.

8. Students in grade 8 are each given one flower seed from a package of mixed flower seeds. The package contains an equal number of daisy, marigold, poppy, and snapdragon seeds. The students roll a four-sided die to find out where each will plant the seed. On the die, 1 means in the front garden at the school, 2 means by the back fence, 3 means in the garden of the senior citizens' home near the school, and 4 means in a flower pot to take home.
 - a) Design a simulation to find the probability that Bianca will plant a marigold in a flower pot. Perform ten trials of the simulation. What is the experimental probability of $P(\text{marigold, flower pot})$?
 - b) Use multiplication to determine the theoretical probability of $P(\text{marigold, flower pot})$.
 - c) Compare your experimental probability with your theoretical probability.

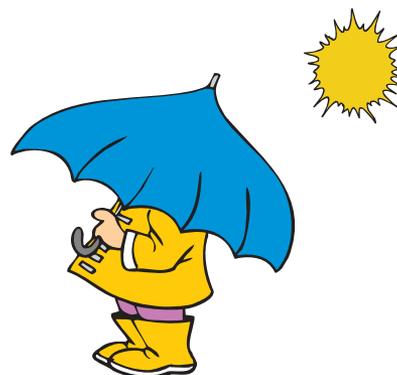
9. Boxes of Oatie Smacks cereal contain a toy racing car in one of five colours: green, purple, black, blue, and red. The likelihood of each colour car is the same, 20%. Trevor uses a five-section spinner to simulate the minimum number of boxes of cereal he will have to buy to make sure he gets at least one car of each colour. The tally chart shows his results.

Green	Purple	Black	Blue	Red

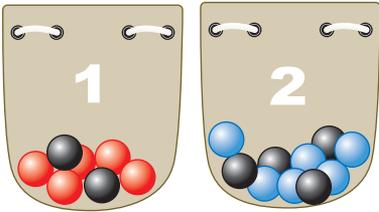
- a) Which car colour was spun last? How do you know?
- b) What is the experimental probability of the spinner landing on blue? Express your answer as a decimal.
- c) What is the theoretical probability of the spinner landing on blue? Express your answer as a decimal.
- d) What is the theoretical probability of getting two blue cars in two consecutive boxes?

Apply

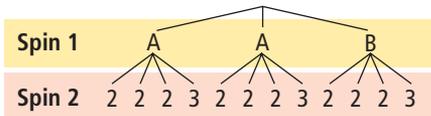
10. The weather forecaster predicts that the chance of rain today is 75% in Victoria and 20% in Calgary. What is the probability that it will rain in both cities today?



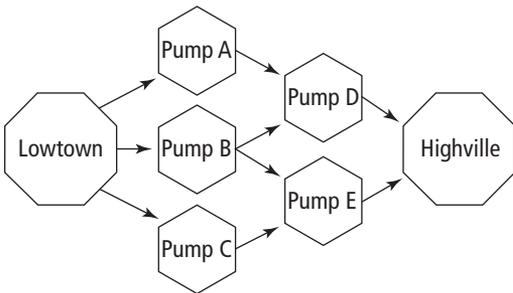
11. What is $P(\text{red, blue})$ if one marble is randomly selected from each bag? Express the answer as a fraction, a decimal, and a percent.



12. The following tree diagram represents the outcomes when two spinners are each spun once.



- a) Draw a picture of both spinners.
- b) What is the probability that the first spinner will land on an A?
- c) What is the probability that the second spinner will land on two?
13. The following diagram shows five water pumping stations between Lowtown and Highville. Water is pumped from Lowtown to Highville through pipes that are connected to the pumping stations as illustrated. With the pumps getting old, the likelihood that a specific pumping station is working at any given time is $\frac{2}{3}$.



- a) In how many different pathways can water be transported from Lowtown to Highville?

- b) How can you use a standard six-sided die to simulate whether a specific pumping station is working?
- c) From the data collected in the table below and the diagram of pumping stations, determine the experimental probability that at least one pathway is available to carry water between the two towns.

Trial #	Pump A	Pump B	Pump C	Pump D	Pump E
1	working	no	working	working	no
2	working	working	working	working	no
3	no	working	no	no	working
4	working	working	no	no	no
5	no	no	working	working	no
6	working	working	no	working	working
7	working	no	working	no	working
8	no	no	no	working	working
9	working	working	working	no	working
10	no	working	no	working	no

Extend

14. It is Random Menu night at the Guess Grill restaurant. You do not order your own meal! For \$3.99 you are given one of four possible appetizers and one of six possible main courses. Jeremy figures that he would be happy with three of the appetizers and three of the main courses.
- a) What is the probability that Jeremy will be happy with both his appetizer and main course?
- b) What is the probability that he will be unhappy with both his appetizer and main course?
- c) Explain why the answers to parts a) and b) do not add to one.

15. The next two batters for the Okotoks Wanderers have batting averages of .313 and .289, respectively. For the first batter, this means that for every 1000 at-bats in the past, he hit the ball and got on base 313 times.
- What is the probability that both players will hit a fair ball and get on base? Express your answer as a decimal to the nearest thousandth.
 - What is the probability that the first player gets a hit and the second player does not? Express your answer as a decimal to the nearest thousandth.
16. From a deck of 52 playing cards, a card is drawn at random. Then the card is placed back in the deck, the deck is shuffled, and a second card is drawn at random. Determine the following probabilities and express each one as a decimal to the nearest thousandth. Consider an ace to be the number one.
- $P(4, 7)$?
 - $P(4, \text{not } 4)$?
 - $P(4, \text{number less than } 4)$?
17. A probability experiment consists of three independent events, A, B, and C. Two of these events have the probabilities $P(A) = \frac{1}{2}$ and $P(B) = \frac{3}{7}$. The probability of all three events occurring is $\frac{9}{70}$. What is the probability of event C, $P(C)$? Express your answer as a fraction and explain your reasoning.

WWW Web Link

Computers are often used to conduct simulations. To try an on-line simulation, go to www.mathlinks8.ca and follow the links.

MATH LINK

- The four sticks are tossed. Two of them land on the table with the decorated side up. The other two fall under the table. What is the theoretical probability that both sticks under the table are decorated side up?
- What if three sticks fall under the table? What is the theoretical probability that all three sticks land decorated side up?
- Set up a simulation to show the experimental probability for part b).

