5.3 Factoring Trinomials

Trinomials with Leading Coefficient of 1

$x^2 + bx + c$

1. Find 2 factors that add to give sum = b and multiply to give product of c. (i, ii)
2. Set up two brackets $(x \quad)(x \quad)$
3. Place the two factors within brackets $(x + i)(x + ii)$

Trinomials with Leading Coefficient other than 1.

$ax^2 + bx + c$

1. Factor out term to simplify
2. Find 2 factors that... add = b
   (i, ii) multiply to give $a \cdot c$
3. Set up brackets $(ax^2 + ix) + (iix + c)$
4. Factor like items from each set of brackets.
5. Simplify equation and verify solution.

Example 1 Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Factor, if possible.

a) $x^2 + 5x + 4$

b) $x^2 + 4x + 6$

c) $x^2 - 29x + 28$

d) $x^2 + 3xy - 18y^2$

a) $+ \to 5$, $\times \to 4$ (1, 4)

$(x + 1)(x + 4)$

$x^2 + 4x + x + 4$

$= x^2 + 5x + 4$

b) $+ \to 4$, $\times \to 6$ none

c) $+ \to -29$, $\times \to +28$ $-1, -28$ $(x - 1)(x - 28)$

d) $+ \to 3$, $\times \to -18$

3, 6

$+6, -3$ $(x + 6y)(x - 3y)$

$x^2 - 3xy + 6xy - 18y^2$

$x^2 + 3xy - 18y^2$

Example 2 Factor Trinomials of the Form $ax^2 + bx + c$, $a \neq 1$

Factor, if possible.

a) $3x^2 + 8x + 4$

b) $6x^2 - 5xy + y^2$

c) $3x^2 + 2x + 4$

d) $(24x^2 - 30x - 9)3$

a) $+ \to 8$, $\times \to 3 \cdot 4 = 12$

2, 6

$(3x^2 + 6x) + (2x + 4)$

$3x(x + 2) + 2(x + 2)$

$(x + 2)(3x + 2)$

$(x + 2)(3x + 2)$

$3x^2 + 2x + 6x + 4$

$3x^2 + 8x + 4$

b) $+ \to -5$, $\times \to 6 \to 6$

$-3, -2$

$(6x^2 - 3xy) + (-2xy + y^2)$

$3x(2x - y) - y(2x - y)$

$(3x - y)(2x - y)$

$(3x - y)(2x - y)$

$6x^2 - 3xy - 2xy + y^2$

$= 6x^2 - 5xy + y^2$

c) $+ \to 2$, $\times \to 3 \cdot 4 = 12$

none

d) -12, $+2$; $+ \to -10$, $\times \to -36$, $= -24$

$3(8x^2 - 10x - 3)$

$3[(8x^2 + 2x) + (-12x - 3)]$

$3[2x(4x + 1) - 3(4x + 1)]$

$3(2x - 3)(4x + 1)$

$3(2x - 3)(4x + 1)$

$3(8x^2 + 2x - 12x - 3)$

$3(8x^2 - 10x - 3)$

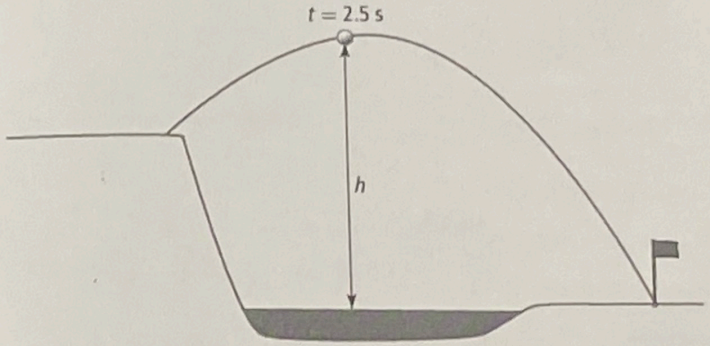
$24x^2 - 30x - 9$

Example 3 Apply Factoring

The world famous *Devil's Cauldron* is the 4th hole at the Banff Springs Golf Course. This is a tough tee shot from an elevated tee that must carry the ball across a glacial lake to a small bowl green. The approximate height of the ball during a typical shot can be represented by the formula $h = -5t^2 + 25t + 30$, where t is the time, in seconds, and h is the height of the ball relative to the green, in metres.

a) Write the formula in factored form.

b) What is the height of the golf ball after 2.5 s?



a) $h = -5t^2 + 25t + 30$

$h = -5(t^2 - 5t - 6)$

$h = -5(t - 6)(t + 1)$

$+ \to -5$, $\times \to -6$

$+1, -6$

b) $-5t^2 + 25t + 30$

$= -5(2.5)^2 + 25(2.5) + 30$

$= 61.25$ m

$-5(t - 6)(t + 1)$

$= -5(2.5 - 6)(2.5 + 1)$

$= 61.25$ m.