Example 1 Multiply or Divide Powers With the Same Base

Write each product or quotient as a power with a single exponent.

a) $(5^{\frac{1}{3}})(5^{\frac{5}{3}})$

$\frac{1}{3} + \frac{5}{3}$

$\frac{6}{3}$

$=2$

$5^2 \rightarrow 25$

b) $(x^5)(x^{-\frac{1}{2}})$

$\frac{5}{1} + -\frac{1}{2}$

$\frac{10}{2} + -\frac{1}{2}$

$x^{\frac{9}{2}}$

c) $\frac{3^{-\frac{3}{4}}}{3^{0.25}}$ -0.75

$-0.75 - 0.25$

$\frac{3}{1}^{-1}$

$= \frac{1}{3}$

d) $\frac{(8)^{1.8}}{16^{0.3}}$ $\frac{(2^3)^{1.8}}{(2^4)^{0.3}}$

$\frac{2^{5.4}}{2^{1.2}}$ $5.4 - 1.2$

$2^{4.2} = 18.3$

Example 2 Simplify Powers With Rational Exponents

Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

a) $(4x^3)^{0.5}$

$4^{0.5}(x^3)^{0.5}$

$4^{\frac{1}{2}} x^{1.5}$

$\sqrt{4}^1$

$= 2x^{1.5}$

b) $\left[(x^3)\left(x^{\frac{3}{2}}\right)\right]^{\frac{1}{2}}$

$(x^{\frac{3}{4}})^{\frac{1}{2}}(x^{\frac{3}{2}})^{\frac{1}{2}}$

$x^{\frac{3}{1}\times\frac{1}{2}}$ $x^{\frac{3}{2}\cdot\frac{1}{2}}$

$x^{\frac{2}{2}\cdot\frac{3}{4}}$

$x^{\frac{6}{4}+\frac{3}{4}}$ $x^{\frac{9}{4}}$

c) $\left(\frac{3^4}{16}\right)^{-0.75}$ $\frac{(3^4)^{-0.75}}{16^{-0.75}} \rightarrow \frac{16^{0.75}}{(3^4)^{0.75}}$

$= \frac{8}{3}$

$= \frac{8}{27}$

Example 3 Apply Powers With Rational Exponents

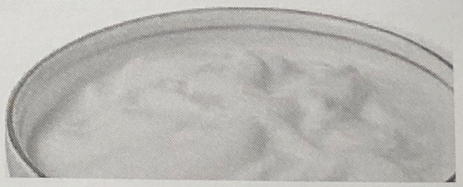
Food manufacturers use a beneficial bacterium called *Lactobacillus bulgaricus* to make yoghurt and cheese. The growth of 10 000 bacteria can be modelled using the formula $N = 10\,000(2)^{\frac{h}{42}}$, where N is the number of bacteria after h hours.

a) What does the value 2 in the formula tell you?

b) How many bacteria are present after 42 h?

c) How many more bacteria are present after 2 h?

d) How many bacteria are present after 105 h?



4.3 Rational Exponents

What do Rational Exponents mean?

$5^{\frac{1}{2}}$

$\sqrt[2]{5^1}$

≈ 2.2

$27^{\frac{1}{3}}$

$\sqrt[3]{27^1}$

$= 3$

$27^{\frac{2}{3}}$

$\sqrt[3]{27^2}$

$\sqrt[3]{729}$

$= 9$

numerator → base

denominator → Root identif

$(\sqrt[3]{27})^2$

$= 9$

Ex 3. a) doubles in population.

b) $N = 10\,000(2)^{\frac{42}{42}}$

$= 10\,000(2)^1$ → 20 000 bacteria

c) $N = 10\,000(2)^{\frac{2}{42}}$

$= 10\,335.5$ bacteria → 335.5 more

d) $N = 10\,000(2)^{\frac{105}{42}}$

$= 56\,568.5$ bacteria