Relating Similar Triangles to Trigonometry

- Remember that similar triangles have equal angles and proportional sides.

- Proportional means equal fractions. So since similar triangles are proportional, you can make equal fractions when comparing corresponding sides.

- Since the triangles have equal angles, the proportional fractions comparing sides would simplify to the same fraction for all triangles with the same angle measurements.

- Those equal simplified comparison fractions are where we get our Trigonometry ratios.

Sine compares opposite to hypotenuse

Cosine compares adjacent to hypotenuse

Tangent compares opposite to adjacent.

SOH CAH TOA

$\text{Sin } S = \frac{O}{H}$ $\text{Cos } C = \frac{A}{H}$ $\text{Tan } T = \frac{O}{A}$

Example 1 Write a Tangent Ratio

Tan = $\frac{\text{opposite}}{\text{Adjacent}}$

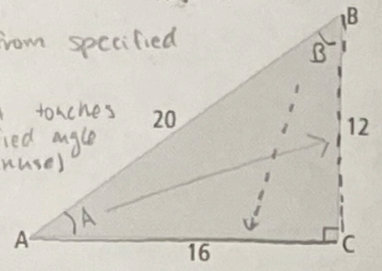
Write each trigonometric ratio.

a) tan A

b) tan B

Opposite → across from specified angle

Adjacent → side that touches the specified angle (not hypotenuse)



a) Tan A = $\frac{\text{opp}}{\text{adj}}$

$= \frac{12}{16}$

$= \frac{3}{4}$

b) Tan B = $\frac{\text{opp}}{\text{adj}}$

$= \frac{16}{12}$

$= \frac{4}{3}$

Example 2 Calculate a Tangent and an Angle

a) Calculate tan 25° to four decimal places.

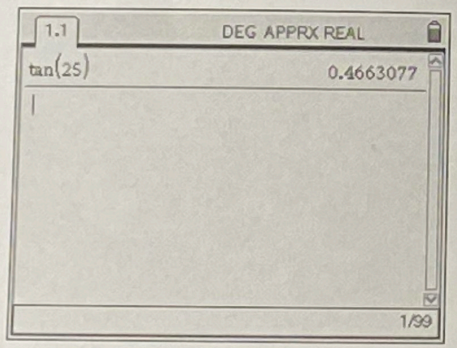
b) Draw a triangle to represent tan θ = $\frac{5}{4}$. Calculate the angle θ to the nearest tenth of a degree.

θ = theta

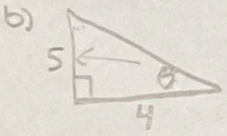
Solution

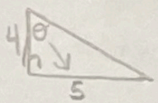
a) ~~tan 25° ≈ 0.4663~~

tan 25° = 0.4663



b)





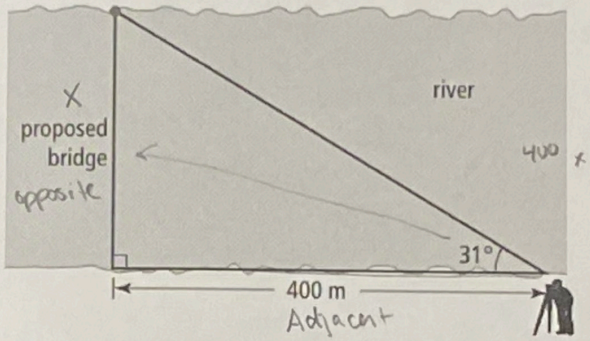
Tan θ = $\frac{\text{opp}}{\text{adj}}$

Tan θ = $\frac{5}{4}$

= 51.3°

Example 3 Determine a Distance Using the Tangent Ratio

A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor uses a theodolite to measure angles. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?



Tan θ = $\frac{\text{opp}}{\text{Adj}}$

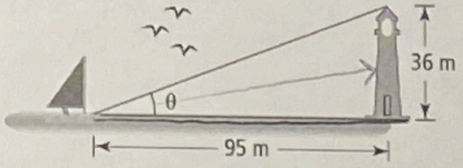
Tan 31° = $\frac{x}{400}$ ×400

240.3 = x

The river is approximately 240m wide.

Example 4 Determine an Angle Using the Tangent Ratio

A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Express your answer to the nearest degree.



Tan θ = $\frac{\text{opp}}{\text{adj}}$

Tan θ = $\frac{36}{95}$

= 20.75°

= 21°